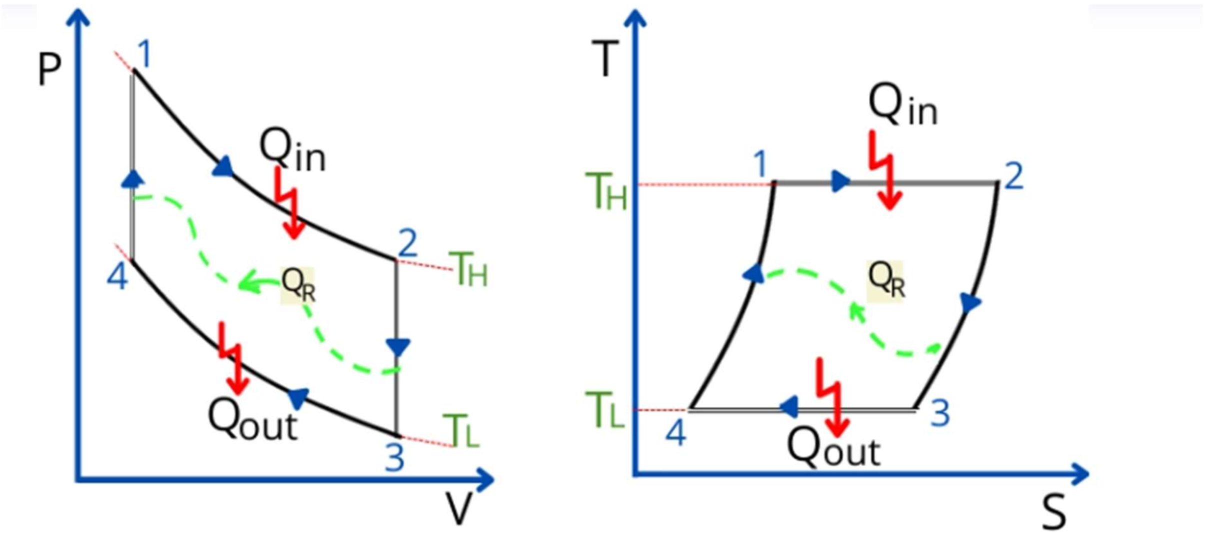
Sterling Cycle

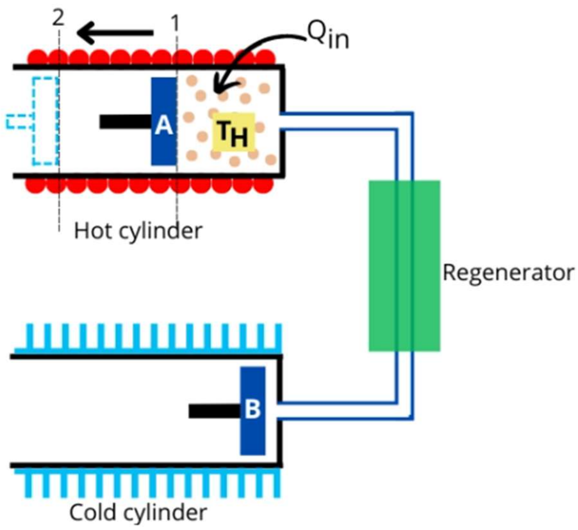


The ideal Stirling cycle consists of the following 4 processes.

- Process 1-2:- Isothermal heat addition/Isothermal expansion
- Process 2-3:- Constant volume heat rejection
- Process 3-4:- Isothermal heat rejection/Isothermal compression
- Process 4-1:- Constant volume heat addition

In the above figure, the Q_R indicates the heat transfer because of the regeneration.
The Stirling cycle consists of two isothermal and two isochoric processes.
Each of the processes is described below:-

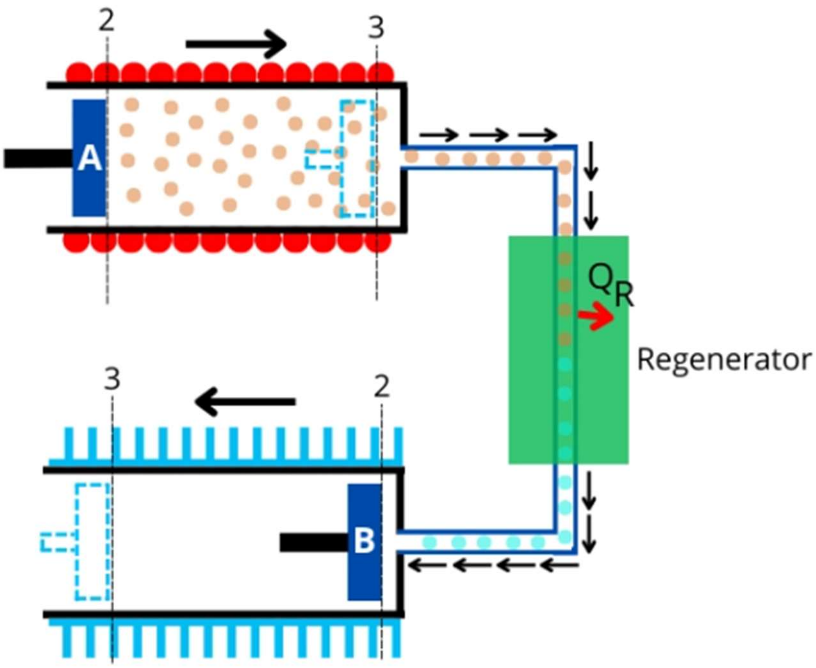
1] Isothermal heat addition:-



The unit shown in the above figure will help to understand the different processes that occur in the Stirling cycle. It consists of a hot cylinder at temperature T_H, a cold cylinder at temperature T_L, and a regenerator.

The process shown in the above figure indicates isothermal heat addition. In this process, the air present in a hot cylinder is heated by an external heat source. During this heat addition, piston-A moves toward the left to cause isothermal expansion of air.

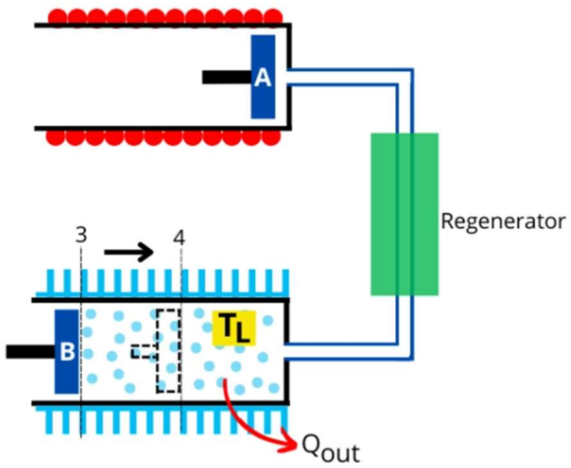
2] Constant volume heat removal:-



During this process, the air in the hot cylinder is transferred to the cold cylinder through the regenerator. For this purpose, piston-A moves to the right while piston-B moves to the left.

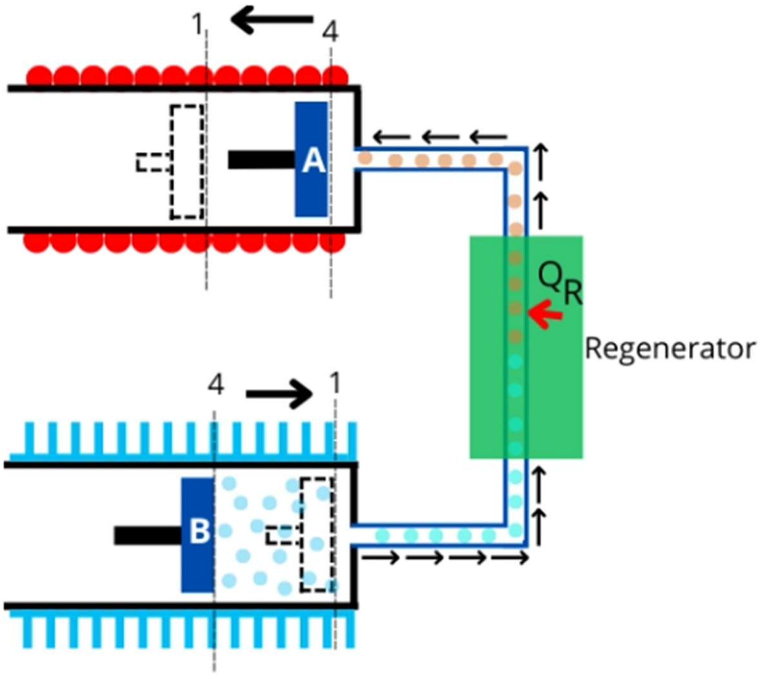
In the regenerator, air rejects the heat to change its temperature from T_H to T_L, and then it enters into the cold cylinder.

3] Isothermal heat rejection/Isothermal compression:-



In this process, the air inside the cold cylinder undergoes isothermal compression with rejecting heat to the external sink. During this process, the piston-B moves inward.

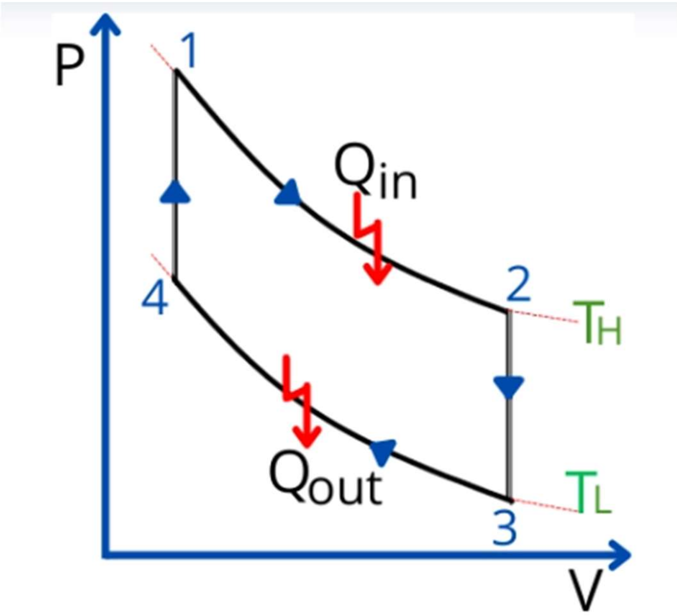
4] Constant volume heat addition:-



During this process, the air inside the cold cylinder is transferred to the hot cylinder through the regenerator.

For this purpose, piston-B moves inward while piston-A moves outward. While passing through the regenerator, the air takes heat Q_R from the regenerator to raise its temperature from T_L to T_H.

Stirling cycle efficiency derivation:



The amount of heat added during the process 1-2 is given by,

$$Q_{\text{in}} = Q_{12} = P_1 v_1 \ln\left(\frac{v_2}{v_1}\right)$$

$$Q_{\text{in}} = RT_1 \ln\left(\frac{v_2}{v_1}\right) \cdots [\because P_1 v_1 = RT_1]$$

$$Q_{\text{in}} = RT_H \ln\left(\frac{v_2}{v_1}\right) \cdots [\because T_1 = T_H]$$

The amount of heat rejected during isothermal process 3-4 is given by,

$$Q_{\text{out}} = Q_{34} = P_3 v_3 \ln\left(\frac{v_4}{v_3}\right)$$

$$Q_{\text{out}} = RT_3 \ln\left(\frac{v_4}{v_3}\right) \cdots [\because P_3 v_3 = RT_3]$$

$$Q_{\text{out}} = RT_L \ln\left(\frac{v_4}{v_3}\right) \cdots [\because T_3 = T_L]$$

The net work done by the cycle is given by,

$$W_{\text{net}} = Q_{\text{net}}$$

$$W_{\text{net}} = Q_{\text{in}} + Q_{\text{out}}$$

$$W_{\text{net}} = RT_H \ln\left(\frac{v_2}{v_1}\right) + RT_L \ln\left(\frac{v_4}{v_3}\right)$$

$$W_{\text{net}} = RT_H \ln\left(\frac{v_2}{v_1}\right) - RT_L \ln\left(\frac{v_3}{v_4}\right)$$

Now the thermal efficiency of the Stirling cycle is given by,

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_{\text{in}}}$$

$$\eta_{\text{th}} = \frac{RT_H \ln\left(\frac{v_2}{v_1}\right) - RT_L \ln\left(\frac{v_3}{v_4}\right)}{RT_H \ln\left(\frac{v_2}{v_1}\right)}$$

From above figure $v_3=v_2$ and $v_4=v_1$

$$\therefore \eta_{\text{th}} = \frac{RT_H \ln\left(\frac{v_2}{v_1}\right) - RT_L \ln\left(\frac{v_2}{v_1}\right)}{RT_H \ln\left(\frac{v_2}{v_1}\right)}$$

$$\eta_{\text{th}} = \frac{T_H - T_L}{T_H}$$

$$\eta_{\text{th}} = 1 - \frac{T_L}{T_H}$$

This is the equation to find the efficiency of the Stirling cycle operating between temperature T_H and T_L.
The above equation is similar to the equation of Carnot cycle efficiency