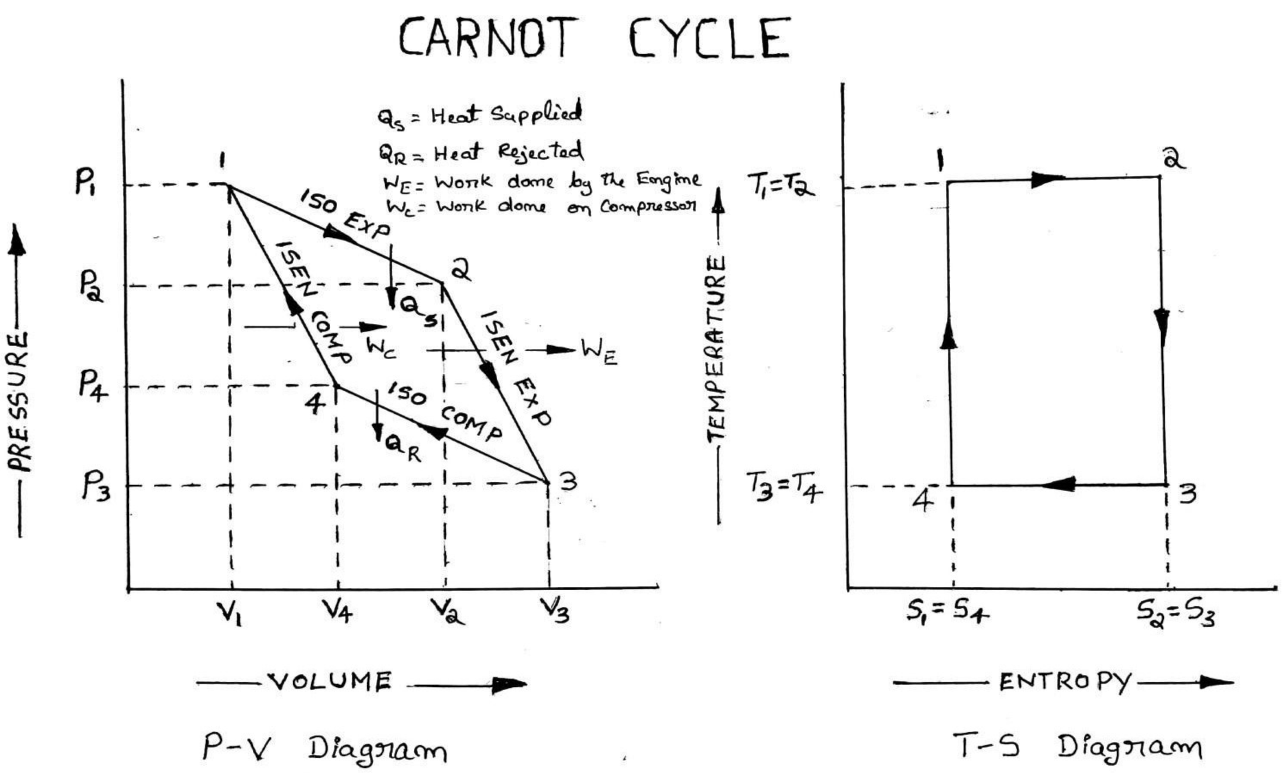
A] CARNOT CYCLE:

Carnot cycle consists of two reversible isothermal and two reversible Isentropic (adiabatic) processes, as shown in below P-V & T-S diagram:



$T_1 = T_{max}$ is the maximum temperature of the cycle

$T_3 = T_{min}$ is the minimum temperature of the cycle

Expansion ratio (process 1-2) & Compression ratio (process 3-4)

Expansion ratio, r = $\frac{V2}{V_1}$ **Compression ratio, r =** $\frac{V3}{V_4}$

Consider 'm' kg of air in a piston cylinder; let P1, V1 & T1 be the initial conditions at state 1.

1] Process (1-2) – Isothermal Expansion:

A hot body at a higher temperature is brought in contact with the bottom of the cylinder. The air expands at a constant temperature, $T_1=T_2$ (T=C)

$\therefore$ Heat Supplied = Work Done by the air during isothermal expansion

i.e. $Q_{1-2} = W_{1-2}$

$\therefore$ $Q_{1-2} = 2.3\ mRT_1 \log [\frac{V_2}{V_1}]$

$\mathbf{Q_{1-2} = 2.3\ mRT_1 \log [r]}$ **----(1)** $\therefore r = \frac{V_2}{V_1}$ = Expansion ratio

2] Process (2-3) – Reversible Adiabatic or Isentropic Expansion:

The hot body is removed and a insulating cap is brought in contact with the bottom of the cylinder. The air expands isentropically.

$\therefore$No heat is supplied or rejected

i.e. $\mathbf{Q_{2-3} = 0}$

3] Process (3-4) – Isothermal Compression:

The insulating cap is removed and a Cold Body is brought in contact with the bottom of the cylinder. The air is compressed at a constant temperature, $T_3=T_4$ (T=C)

Heat Rejected = Work done on the air during isothermal compression

i.e. $Q_{3-4} = W_{3-4}$

$\therefore\ Q_{3-4} = 2.3\ mRT_3 \log [\frac{V_3}{V_4}]$

$\mathbf{Q_{3-4} = 2.3\ mRT_3 \log [r]}$ **----(2)** $\therefore\ r = \frac{V_3}{V_4}$ = Compression ratio

4] Process (4-1) – Reversible Adiabatic or Isentropic Compression:

The cold body is removed and an insulating cap is brought in contact with the bottom of the cylinder. The air compressed isentropically.

$\therefore$ No heat is supplied or rejected

i.e. $\mathbf{Q_{4-1} = 0}$

<u>Work Done in Carnot Engine</u> (W):

Work Done = Heat Supplied – Heat Rejected

i.e W = $Q_{1-2} - Q_{3-4}$

$\therefore$ $\mathbf{W = 2.3\ mRT_1 \log [r] - 2.3\ mRT_3 \log [r]}$ **-------------- (3)**

<u>Efficiency of Carnot Engine</u> (η):

Efficiency, $\eta = \frac{\text{Work Done}}{\text{Heat Supplied}}$

$$\eta = \frac{W}{Q_{1-2}}$$

i.e. $$\eta = \frac{2.3\ mRT_1 \log [r] - 2.3\ mRT_3 \log [r]}{2.3\ mT_1 \log [r]}$$

$$\eta = \frac{2.3\ mR\log [r]\ (T_1 - T_3)}{2.3\ mR \log [r]\ T_1}$$

$$\eta = \frac{T_1 - T_3}{T_1}$$

$$\eta = \frac{T1}{T_1} - \frac{T3}{T_1}$$

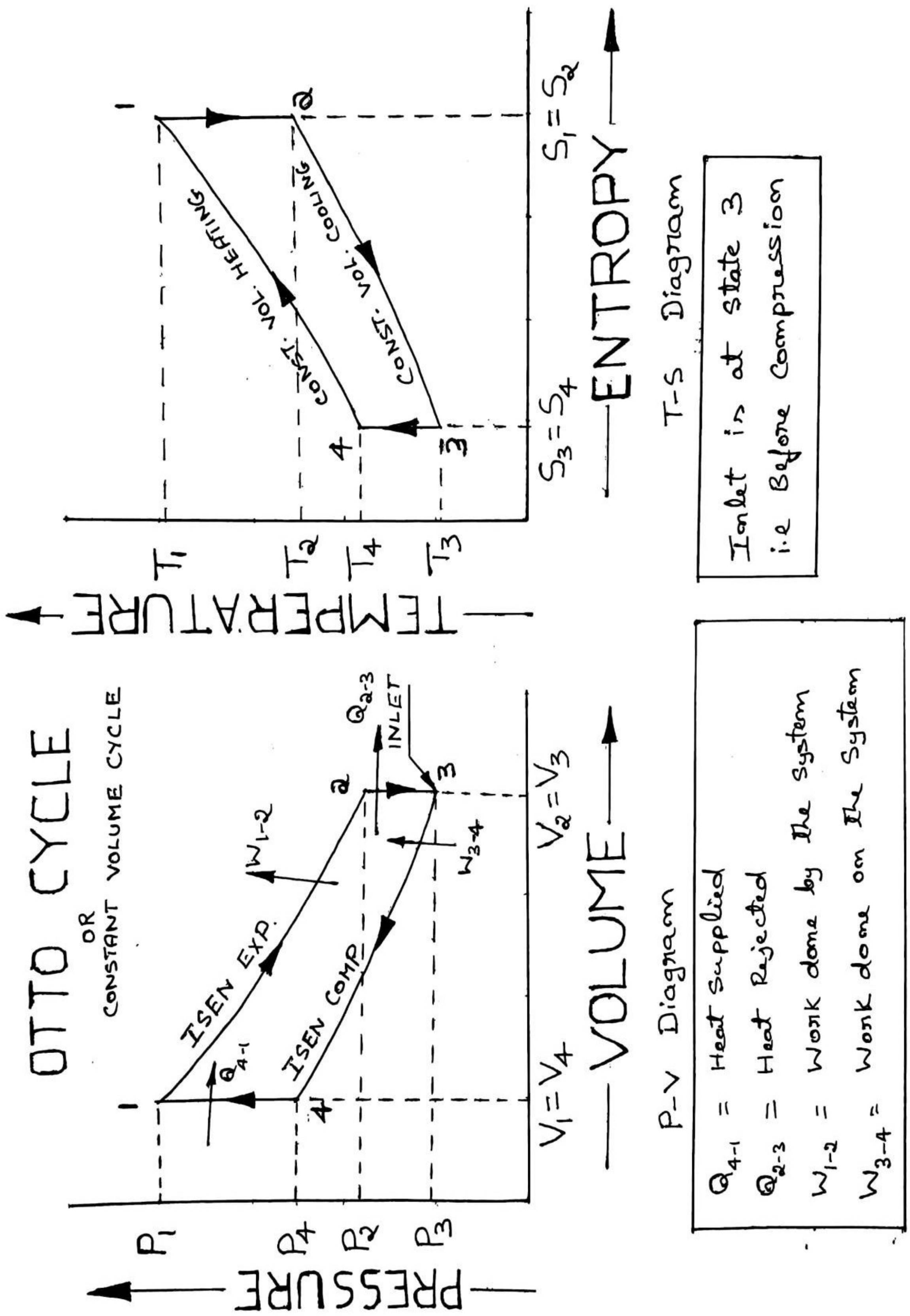
$$\eta = 1 - \frac{T3}{T_1}$$

or

$$\eta = 1 - \frac{Tmin}{T_{max}}$$ is the Efficiency of Carnot Engine

B] <u>OTTO CYCLE</u> or <u>CONSTANT VOLUME CYCLE</u>

Otto cycle consists of two reversible constant volume and two reversible Isentropic (adiabatic) processes, as shown in below P-V & T-S diagram:



Expansion Ratio , $r = \frac{V_2}{V_1}$ [Process (1-2)]

Compression Ratio , $r = \frac{V_3}{V_4}$ [Process (3-4)]

$\frac{\text{Higher Volume}}{\text{Lower Volume}}$

Consider 'm' kg of air in an otto cycle engine Cylinder with initial Conditions $P_1 V_1$ and T_1 at state (1).

Process (1-2) → Reversible Adiabatic or Isentropic Expansion

Here Hot body is removed and insulating cap is brought in contact with the bottom of the cylinder

∴ No Heat is supplied or Rejected

(Rest part of cylinder is completely insulated)

∴ $\boxed{Q_{1-2} = 0}$

Process (2-3) → Constant Volume Cooling

Here Insulating cap is removed and cold Body is brought in contact with the Bottom of the cylinder.

∴ Heat is Rejected at constant volume

i.e Heat Rejected , $\boxed{Q_{2-3} = m C_v [T_2 - T_3]}$

[Higher Temperature - Lower Temperature
Refer T-S Diagram]

Process (3-4) → Reversible Adiabatic or Isentropic Compression

Here Cold Body is removed and insulating cap is brought in contact with the Bottom of the cylinder.

∴ No Heat is supplied or Rejected

(Rest Part of cylinder is completely insulated)

∴ $\boxed{Q_{3-4} = 0}$

Process (4-1) → Constant Volume Heating

Here Insulating cap is removed and Hot Body is brought in contact with the bottom of the cylinder

∴ Heat is supplied at a constant volume

Heat supplied, $\boxed{Q_{4-1} = mC_v (T_1 - T_4)}$

[Higher Temperature – Lower Temperature
Refer T-S Diagram]

Work Done during otto cycle

Work Done = Heat Supplied – Heat Rejected

i.e $W = Q_{4-1} - Q_{2-3}$

$$\boxed{W = mC_v (T_1 - T_4) - mC_v (T_2 - T_3)}$$

Efficiency of otto cycle :-

$$\text{Efficiency} = \frac{\text{Work Done}}{\text{Heat supplied}}$$

i.e $\eta = \dfrac{W}{Q_{4-1}}$

$$\eta = \frac{mC_v (T_1 - T_4) - mC_v (T_2 - T_3)}{mC_v (T_1 - T_4)}$$

$$\eta = \cancel{\frac{mC_v (T_1 - T_4)}{mC_v (T_1 - T_4)}} - \frac{\cancel{mC_v}(T_2 - T_3)}{\cancel{mC_v}(T_1 - T_4)}$$

$$\boxed{\eta = 1 - \frac{(T_2 - T_3)}{(T_1 - T_4)}} \quad --- (1)$$

In above eq (1) Replace T_1 and T_4 using Process (1-2) and process (3-4)

Process (1-2)
Isentropic Expansion $[PV^{r} = C]$
W.K.T

a) $P_1V_1^{r} = P_2V_2^{r} = PV^{r}$

b) $\frac{P_1}{P_2} = \left[\frac{V_2}{V_1}\right]^{r}$

c) $\frac{T_1}{T_2} = \left[\frac{V_2}{V_1}\right]^{r-1}$

d) $\frac{T_1}{T_2} = \left[\frac{P_1}{P_2}\right]^{\frac{r-1}{r}}$

Process (3-4)
Isentropic Compression $[PV^{r} = C]$
W.K.T

a) $P_3V_3^{r} = P_4V_4^{r} = PV^{r}$

b) $\frac{P_3}{P_4} = \left[\frac{V_4}{V_3}\right]^{r}$

c) $\frac{T_3}{T_4} = \left[\frac{V_4}{V_3}\right]^{r-1}$

d) $\frac{T_3}{T_4} = \left[\frac{P_3}{P_4}\right]^{\frac{r-1}{r}}$

let us take eqn (C) from Process (1-2)

i.e c) $\frac{T_1}{T_2} = \left[\frac{V_2}{V_1}\right]^{r-1}$

i.e $\frac{T_1}{T_2} = [r]^{r-1}$ $\left[\because \frac{V_2}{V_1} = r = \text{Expansion Ratio}\right]$

$$\therefore \boxed{T_1 = T_2 \times [r]^{r-1}} \quad \text{----(2)}$$

also let us take eqn (C) from Process (3-4)

i.e c) $\frac{T_3}{T_4} = \left[\frac{V_4}{V_3}\right]^{r-1}$

$\therefore \frac{T_3}{T_4} = \frac{1}{[r]^{r-1}}$

Here $\frac{V_3}{V_4} = r$ = Compression Ratio

$\therefore \frac{V_4}{V_3} = \frac{1}{r}$

$\left[\frac{V_4}{V_3}\right]^{r-1} = \frac{1}{[r]^{r-1}}$

$$\boxed{\frac{T_3}{T_4} = \frac{1}{[r]^{\gamma-1}}} \quad \text{-----} (3)$$

Let us take This as a new eqn (3)

i.e $T_3 = \dfrac{T_4}{[r]^{\gamma-1}}$

$$T_3 \times [r]^{\gamma-1} = T_4$$

or $$\boxed{T_4 = T_3 \times [r]^{\gamma-1}} \quad \text{-----} (4)$$

Now substituting eqn (2) and eqn (4) in eqn (1)

We get $$\eta = 1 - \frac{T_2 - T_3}{T_1 - T_4}$$

Replace T_1 and T_4 using eqn (2) and (4)

i.e $$\eta = 1 - \frac{T_2 - T_3}{T_2[r]^{\gamma-1} - T_3[r]^{\gamma-1}}$$

$$\eta = 1 - \frac{\cancel{T_2 - T_3}}{[r]^{\gamma-1}\cancel{(T_2 - T_3)}}$$

$$\therefore \boxed{\eta = 1 - \frac{1}{[r]^{\gamma-1}}} \quad \text{---} (5)$$

Is the Efficiency of otto Cycle in terms of Compression Ratio

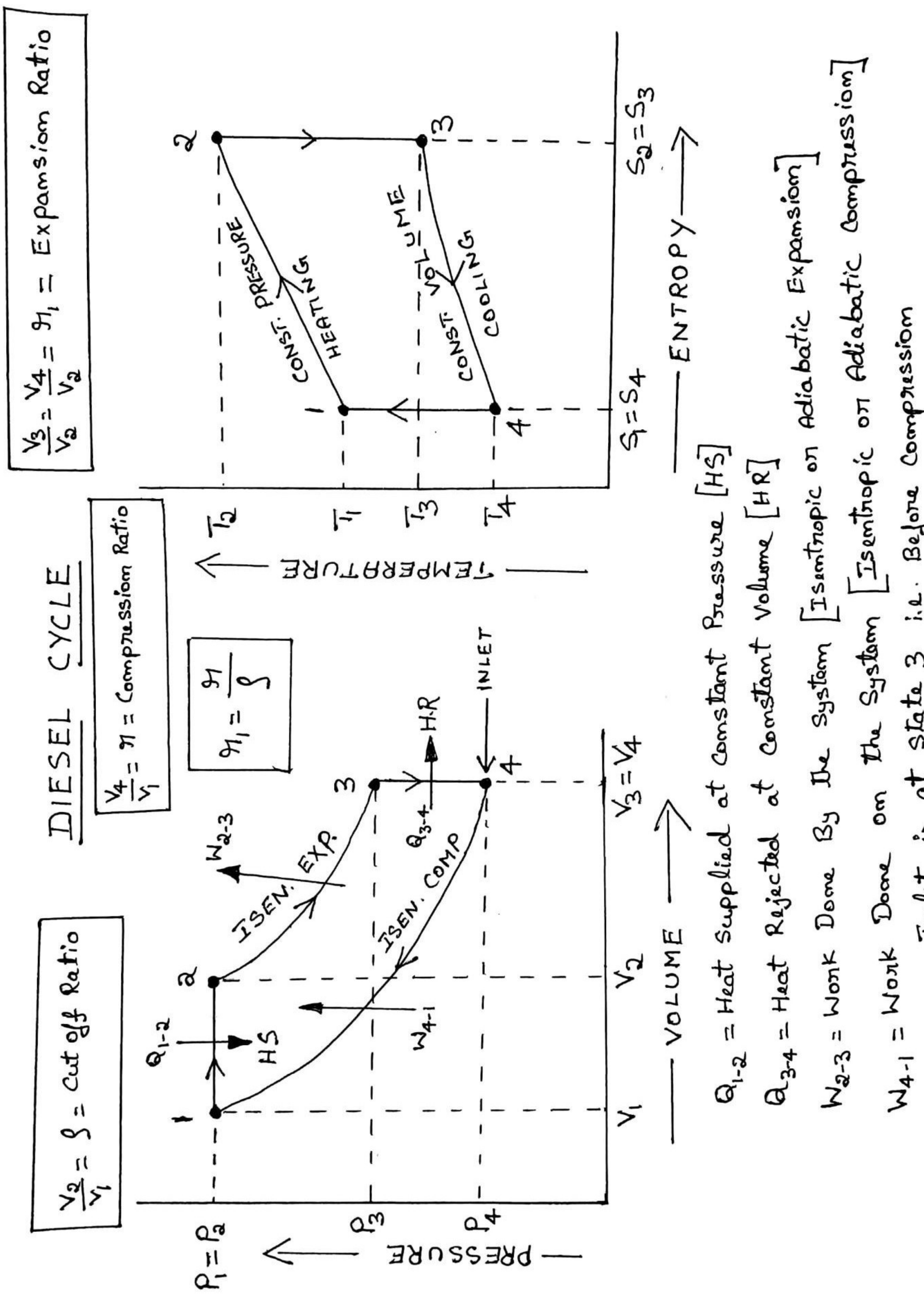
$$\boxed{\eta = 1 - \frac{T_3}{T_4}} \quad \text{---} (6)$$

$$\left[\begin{array}{c}\text{From eqn (3)} \\ \dfrac{1}{[r]^{\gamma-1}} = \dfrac{T_3}{T_4}\end{array}\right]$$

Is the Efficiency of otto Cycle in terms of Temperatures

C] <u>DIESEL CYCLE</u> or <u>CONSTANT PRESSURE CYCLE</u>

Diesel cycle consists of one reversible Constant Pressure Process, One reversible Constant volume process and two reversible Isentropic (adiabatic) processes, as shown in below P-V & T-S diagram:



Let us consider 'm' Kg of air in a Diesel cycle engine Cylinder. Let $P_1 V_1$ and T_1 be the initial conditions at state 1.

Process (1-2) → Constant Pressure Heating $P = C$

Insulating cap is removed and a Hot Body is brought in Contact with the Bottom of the cylinder.

Heat is supplied at a constant Pressure, the temperature Rises from T_1 to T_2.

∴ Heat supplied, $\boxed{Q_{1-2} = m c_p (T_2 - T_1)}$

Process (2-3) → Isentropic or adibatic Expansion $PV^r = C$

Hot Body is removed and an insulating Cap is brought in contact with the bottom of the cylinder.

Air is expanded Isentropically from V_2 to V_3.

No Heat is added or Rejected

∴ $\boxed{Q_{2-3} = 0}$ Temperature decreases from T_2 to T_3

Process (3-4) → Constant Volume cooling $V = C$

Insulating Cap is removed and a cold Body is brought in contact with the bottom of the cylinder.

Heat is rejected at a constant Volume, the temperature decreases from T_3 to T_4

∴ Heat Rejected, $\boxed{Q_{3-4} = m c_v (T_3 - T_4)}$

Process (4-1) → Isentropic or adibatic Compression $PV^r = C$

Cold Body is removed and an insulating Cap is brought in contact with the bottom of the cylinder.

Air is Compressed Isentropically from V_4 to V_1

No Heat is added or Rejected

∴ $\boxed{Q_{4-1} = 0}$ Temperature increases from T_4 to T_1

Work Done By the Diesel Cycle Engine [W]

Work Done = Heat Supplied - Heat Rejected

i.e $W = Q_{1-2} - Q_{3-4}$

$$\boxed{W = mC_p(T_2 - T_1) - mC_v(T_3 - T_4)}$$

Efficiency of Diesel cycle [η]

$$\text{Efficiency} = \frac{\text{Work Done}}{\text{Heat Supplied}}$$

i.e $$\eta = \frac{W}{Q_{1-2}}$$

$$\eta = \frac{mC_p(T_2 - T_1) - mC_v(T_3 - T_4)}{mC_p(T_2 - T_1)}$$

$$\eta = \cancel{\frac{mC_p(T_2 - T_1)}{mC_p(T_2 - T_1)}} - \frac{\cancel{m}C_v(T_3 - T_4)}{\cancel{m}C_p(T_2 - T_1)}$$

$$\eta = 1 - \frac{C_v}{C_p}\frac{(T_3 - T_4)}{(T_2 - T_1)}$$

W.K.T $\frac{C_p}{C_v} = \gamma \quad \therefore \frac{C_v}{C_p} = \frac{1}{\gamma}$

$$\therefore \boxed{\eta = 1 - \frac{1}{\gamma}\frac{(T_3 - T_4)}{(T_2 - T_1)}} \quad ---(1)$$

In the above equation replace T_1, T_2 and T_3 interms of T_4

Cut off Ratio, $\boxed{\rho = \frac{V_2}{V_1}}$ Compression Ratio, $\boxed{r = \frac{V_4}{V_1}}$

Expansion Ratio, $\boxed{r_1 = \frac{V_3}{V_2} = \frac{V_4}{V_2}}$ $\left[\because V_3 = V_4\right]$

Now let us take $r_1 = \frac{V_4}{V_2}$ Here multiply and Divide by V_1

We get $r_1 = \frac{V_4}{V_2} \times \frac{V_1}{V_1}$

Let us rearrange as $r_1 = \frac{V_4}{V_1} \times \frac{V_1}{V_2}$

i.e $r_1 = r \times \frac{1}{\rho}$ $\because \frac{V_4}{V_1} = r$ and $\frac{V_2}{V_1} = \rho$

i.e $\frac{V_1}{V_2} = \frac{1}{\rho}$

$\therefore \boxed{r_1 = \frac{r}{\rho}}$ - - - (2)

i.e Expansion Ratio $= \dfrac{\text{Compression Ratio}}{\text{Cut off Ratio}}$

Let us consider Process (1-2) Constant Pressure Heating using general gas equation

i.e $\frac{\cancel{P_1} V_1}{T_1} = \frac{\cancel{P_2} V_2}{T_2}$ $\because P_1 = P_2$ i.e $P = C$

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ now rearrange for T_2

i.e $\frac{T_2}{T_1} = \frac{V_2}{V_1}$

$\therefore T_2 = T_1 \times \frac{V_2}{V_1}$

$\boxed{\text{i.e } T_2 = T_1 \times \rho}$ - - - (3) $\because \frac{V_2}{V_1} = \rho$, cut off Ratio

Let us consider Process (2-3) Isentropic Expansion $PV^{\gamma} = C$

W.K.T for (2-3)

(a) $P_2 V_2^{\gamma} = P_3 V_3^{\gamma} = PV^{\gamma}$

(b) $\frac{P_2}{P_3} = \left[\frac{V_3}{V_2}\right]^{\gamma}$

(c) $\frac{T_2}{T_3} = \left[\frac{V_3}{V_2}\right]^{\gamma-1}$

(d) $\frac{T_2}{T_3} = \left[\frac{P_2}{P_3}\right]^{\frac{\gamma-1}{\gamma}}$

Using equation (c) i.e $\frac{T_2}{T_3} = \left[\frac{V_3}{V_2}\right]^{\gamma-1}$

$\therefore \frac{T_2}{T_3} = \left[r_1\right]^{\gamma-1}$ $\because \frac{V_3}{V_2} = r_1$ = Expansion Ratio

Rearrange the above equation for T_3

i.e $T_2 = T_3 \times \left[r_1\right]^{\gamma-1}$

or $T_3 = \frac{T_2}{\left[r_1\right]^{\gamma-1}}$

i.e $T_3 = \frac{T_1 \times \rho}{\left[r_1\right]^{\gamma-1}}$ $\because T_2 = T_1 \times \rho$ from eq'n (3)

$\therefore T_3 = \frac{T_1 \times \rho}{\left[\frac{r}{\rho}\right]^{\gamma-1}}$ $\because r_1 = \frac{r}{\rho}$ from eq'n (2)

$\therefore \left[r_1\right]^{\gamma-1} = \left[\frac{r}{\rho}\right]^{\gamma-1}$

i.e $T_3 = \frac{T_1 \times \rho}{\frac{\left[r\right]^{\gamma-1}}{\left[\rho\right]^{\gamma-1}}}$

$$T_3 = \frac{T_1 \times \rho \times \left[\rho\right]^{\gamma-1}}{\left[r\right]^{\gamma-1}}$$

i.e $T_3 = \dfrac{T_1 \rho^1 \times \rho^{\gamma-1}}{[r]^{\gamma-1}}$

$T_3 = \dfrac{T_1 \rho^{1+\gamma-1}}{[r]^{\gamma-1}}$

$\boxed{\rho^{\cancel{1}+\gamma-\cancel{1}} = \rho^\gamma}$

$$\boxed{T_3 = \frac{T_1 \rho^\gamma}{[r]^{\gamma-1}}} \quad ---- (4)$$

Let us consider Process (4-1) Isentropic compression $PV^\gamma = C$

W.K.T for (4-1)

(a) $P_4 V_4^\gamma = P_1 V_1^\gamma = PV^\gamma$

(b) $\dfrac{P_4}{P_1} = \left[\dfrac{V_1}{V_4}\right]^\gamma$

(c) $\dfrac{T_4}{T_1} = \left[\dfrac{V_1}{V_4}\right]^{\gamma-1}$

(d) $\dfrac{T_4}{T_1} = \left[\dfrac{P_4}{P_1}\right]^{\frac{\gamma-1}{\gamma}}$

Using equation (c) i.e $\dfrac{T_4}{T_1} = \left[\dfrac{V_1}{V_4}\right]^{\gamma-1}$

$\therefore \dfrac{T_4}{T_1} = \dfrac{1}{[r]^{\gamma-1}}$

$\because \dfrac{V_4}{V_1} = r =$ Compression ratio

i.e $\dfrac{V_1}{V_4} = \dfrac{1}{r}$

$\left[\dfrac{V_1}{V_4}\right]^{\gamma-1} = \dfrac{1}{[r]^{\gamma-1}}$

Rearrange the above equation for T_1

i.e $T_4 = \dfrac{T_1}{[r]^{\gamma-1}}$

or $$\boxed{T_1 = T_4 \times [r]^{\gamma-1}} \quad --- (5)$$

Let us take equation (3)

$T_2 = T_1 \times \rho$ Now replace T_1 using eq'n (5)

i.e $\boxed{T_2 = T_4 \times [r]^{\gamma-1} \times \rho}$ ---- (6) $\because T_1 = T_4 \times [r]^{\gamma-1}$

Let us take equation (4)

$$T_3 = \frac{T_1 \rho^{\gamma}}{[r]^{\gamma-1}}$$ Now replace T_1 using eq'n (5)

$$\text{i.e } T_3 = \frac{T_4 \times \cancel{[r]^{\gamma-1}} \times \rho^{\gamma}}{\cancel{[r]^{\gamma-1}}}$$ $\because T_1 = T_4 \times [r]^{\gamma-1}$

$\boxed{T_3 = T_4 \times \rho^{\gamma}}$ ---- (7)

Now substitute equation (5), (6) and (7) in eq'n (1)

i.e Replace T_1, T_2 and T_3 from equation (1)

$$\text{i.e } \eta = 1 - \frac{1}{\gamma}\frac{(T_3 - T_4)}{(T_2 - T_1)}$$

$$\therefore \eta = 1 - \frac{1}{\gamma}\frac{(T_4 \times \rho^{\gamma} - T_4)}{(T_4 [r]^{\gamma-1} \rho - T_4 \times [r]^{\gamma-1})}$$

$$\eta = 1 - \frac{1}{\gamma}\frac{\cancel{T_4}(\rho^{\gamma} - 1)}{\cancel{T_4} \times [r]^{\gamma-1}(\rho - 1)}$$

$$\eta = 1 - \frac{1}{\gamma} \frac{(\rho^{\gamma} - 1)}{[r]^{\gamma-1}(\rho - 1)}$$

or

$$\boxed{\eta = 1 - \frac{1}{[r]^{\gamma-1}} \left[\frac{(\rho^{\gamma} - 1)}{\gamma(\rho - 1)} \right]} \quad \text{---} (8)$$

Is the efficiency for Diesel cycle

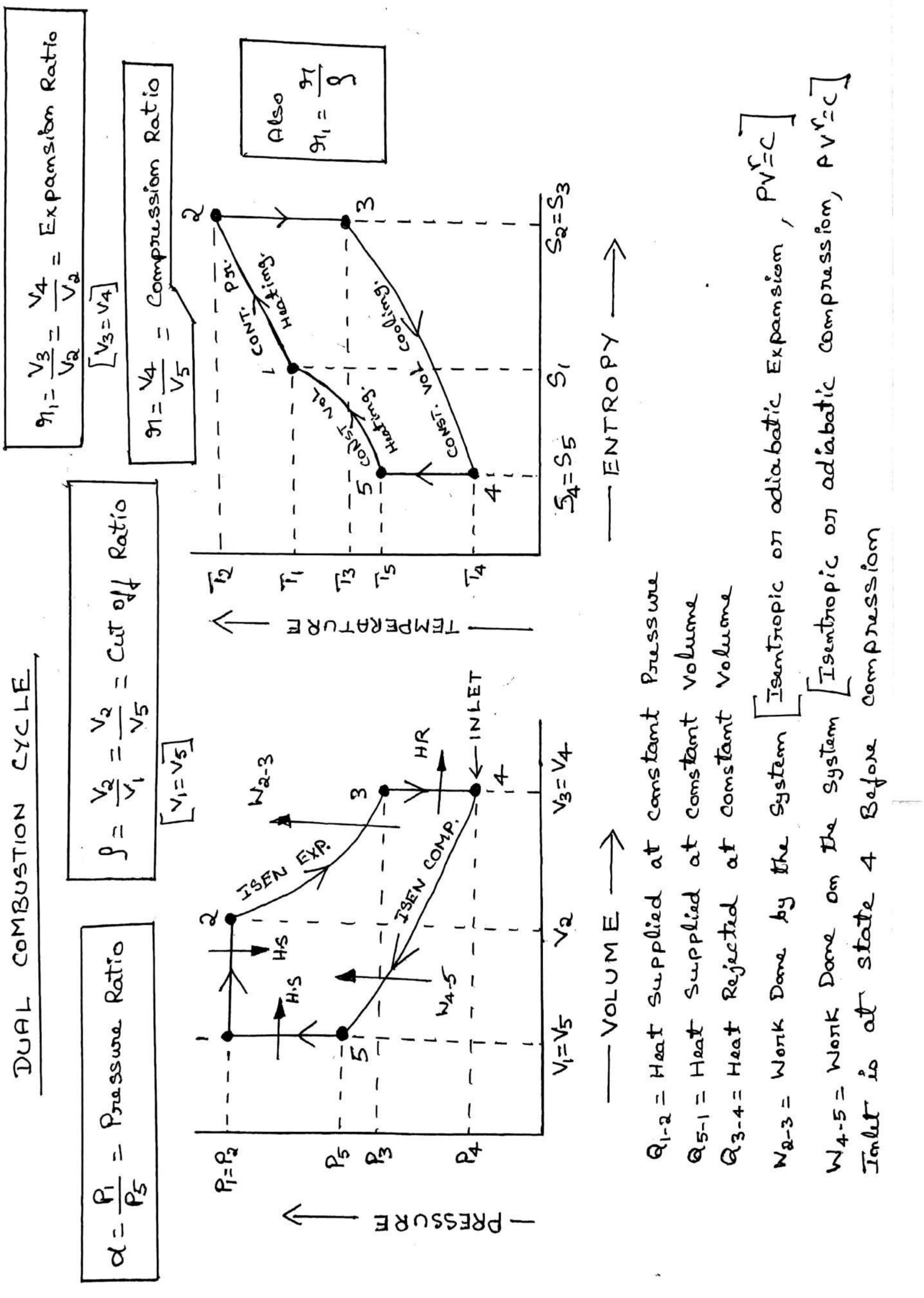
Here, r = Compression Ratio

ρ = Cut off Ratio

$\gamma = \frac{C_p}{C_v}$ i.e Specific Heat Ratio

D] DUAL COMBUSTION CYCLE:

Dual combustion cycle consists of one reversible Constant Pressure Process, Two reversible Constant volume process and two reversible Isentropic (adiabatic) processes, as shown in below P-V & T-S diagram:



It is a combination of otto and diesel cycle, it is also known as semi diesel cycle.

Let us consider 'm' Kg of air inside Dual combustion cycle cylinder. Let $P_1 V_1$ and T_1 be the initial conditions at state 1.

Process (1-2) Constant Pressure Heating :- $[P = C]$ $[P_1 = P_2]$

Heat supplied at constant Pressure, temperature increases from T_1 to T_2.

Heat Supplied $\boxed{Q_{1-2} = m C_p (T_2 - T_1)}$

[$T_2 - T_1$ → Higher Temperature – Lower Temperature, Refer T-S diagram]

Process (2-3) Isentropic or Adiabatic Expansion $[PV^{\gamma} = C]$

No Heat is Supplied or Rejected, Temperature decreases from T_2 to T_3

i.e $\boxed{Q_{2-3} = 0}$

Process (3-4) Constant Volume Cooling $[V = C]$ $[V_3 = V_4]$

Heat is Rejected at Constant Volume, Temperature decreases from T_3 to T_4

Heat Rejected, $\boxed{Q_{3-4} = m C_v (T_3 - T_4)}$

[$T_3 - T_4$ → Higher Temp. – Lower Temp., Refer T-S diagram]

Process (4-5) Isentropic or Adiabatic Compression $[PV^{\gamma} = C]$

No heat is supplied or Rejected, Temperature increases from T_4 to T_5

i.e $\boxed{Q_{4-5} = 0}$

Process (5-1) constant Volume Heating $[V=C]$ $[V_1=V_5]$

Heat supplied at Constant Volume, Temperature increases from T_5 to T_1

Heat supplied, $\boxed{Q_{5-1} = mC_v(T_1-T_5)}$

[T_1-T_5 → Higher Temp. — Lower Temp., Refer T-S diagram]

Note:-

→ Heat is supplied by getting a Hot Body in contact with the bottom of Cylinder — —

→ Heat is rejected by getting a Cold Body in Contact with The Bottom of Cylinder

→ Isentropic or adibatic Compression and Expansion takes place by getting a insulating cap in contact with The bottom of Cylinder.

→ In dual combustion cycle, Heat is supplied in Two stages Q_{5-1} and Q_{1-2}

Work Done during Dual Combustion cycle Engine $[W]$

Work Done = Heat Supplied — Heat Rejected

i.e $$W = [Q_{1-2} + Q_{5-1}] - [Q_{3-4}]$$

$$W = [mC_p(T_2-T_1) + mC_v(T_1-T_5)] - [mC_v(T_3-T_4)]$$

Efficiency of Dual combustion cycle $[\eta]$

$$\text{Efficiency} = \frac{\text{Work Done}}{\text{Heat Supplied}} = \frac{\text{Heat supplied} - \text{Heat Rejected}}{\text{Heat Supplied}}$$

i.e. $$\eta = \frac{W}{[Q_{1-2}+Q_{5-1}]}$$

$$\eta = \frac{[Q_{1-2} + Q_{5-1}] - [Q_{3-4}]}{Q_{1-2} + Q_{5-1}}$$

$$\eta = \frac{[mC_p(T_2-T_1) + mC_v(T_1-T_5)] - [mC_v(T_3-T_4)]}{mC_p(T_2-T_1) + mC_v(T_1-T_5)}$$

$$\eta = \frac{\cancel{mC_p(T_2-T_1) + mC_v(T_1-T_5)}}{\cancel{mC_p(T_2-T_1) + mC_v(T_1-T_5)}} - \frac{mC_v(T_3-T_4)}{mC_p(T_2-T_1) + mC_v(T_1-T_5)}$$

$$\eta = 1 - \frac{\cancel{m}\, C_v(T_3-T_4)}{\cancel{m}[C_p(T_2-T_1) + C_v(T_1-T_5)]}$$

$$\eta = 1 - \frac{\frac{\cancel{C_v}}{\cancel{C_v}}(T_3-T_4)}{\frac{C_p}{C_v}(T_2-T_1) + \frac{\cancel{C_v}}{\cancel{C_v}}(T_1-T_5)}$$ [Divide by C_v]

$\frac{C_p}{C_v} = \gamma$

$$\boxed{\eta = 1 - \frac{(T_3-T_4)}{\gamma(T_2-T_1) + (T_1-T_5)}} \quad --- (1)$$

Also from P–V diagram

Pressure Ratio, $\boxed{\alpha = \frac{P_1}{P_5}}$

Cut-off Ratio, $\boxed{\rho = \frac{V_2}{V_1} = \frac{V_2}{V_5}}$ $[\because V_1 = V_5]$

Compression ratio, $\boxed{r = \frac{V_4}{V_5} = \frac{V_3}{V_1}}$ $\left[\because \begin{array}{l} V_4 = V_3 \\ V_5 = V_1 \end{array}\right]$

Expansion Ratio, $\boxed{r_1 = \frac{V_3}{V_2} = \frac{V_4}{V_2}}$ $[\because V_3 = V_4]$

take $r_1 = \frac{V_3}{V_2}$ multiply and divide by V_1

i.e $r_1 = \frac{V_3}{V_2} \times \frac{V_1}{V_1}$ on Rearranging $\boxed{r_1 = \frac{V_3}{V_1} \times \frac{V_1}{V_2}}$

$\because \frac{V_3}{V_1} = r$

$\therefore r_1 = r \times \frac{1}{\rho}$ or $\boxed{r_1 = \frac{r}{\rho}}$

$\frac{V_2}{V_1} = \rho \therefore \frac{V_1}{V_2} = \frac{1}{\rho}$

From Process (1-2) Constant Pressure Heating. [P=C]

using general gas equation $\frac{\cancel{P_1}V_1}{T_1} = \frac{\cancel{P_2}V_2}{T_2}$ $[\because P_1 = P_2]$

$\therefore \frac{V_1}{T_1} = \frac{V_2}{T_2}$

$\therefore \frac{T_2}{T_1} = \frac{V_2}{V_1}$ or $T_2 = T_1 \times \frac{V_2}{V_1}$ i.e $\boxed{T_2 = T_1 \times \rho}$ --- (2)

From Process (2-3) Isentropic Expansion $PV^\gamma = C$

Using eq^n (c) $\frac{T_2}{T_3} = \left[\frac{V_3}{V_2}\right]^{\gamma-1}$

W.K.T (a) $P_2V_2^\gamma = P_3V_3^\gamma = PV^\gamma$

(b) $\frac{P_2}{P_3} = \left[\frac{V_3}{V_2}\right]^\gamma$

(c) $\frac{T_2}{T_3} = \left[\frac{V_3}{V_2}\right]^{\gamma-1}$

(d) $\frac{T_2}{T_3} = \left[\frac{P_2}{P_3}\right]^{\frac{\gamma-1}{\gamma}}$

i.e $T_2 = T_3 \times [r_1]^{\gamma-1}$ $\boxed{\because \frac{V_3}{V_2} = r_1}$

or $T_3 = \frac{T_2}{[r_1]^{\gamma-1}}$

$T_3 = \frac{T_1 \times \rho}{\left[\frac{r}{\rho}\right]^{\gamma-1}}$ $\because T_2 = T_1 \times \rho$ from eq^n(2)

$r_1 = \frac{r}{\rho}$

i.e $T_3 = \frac{T_1 \times \rho}{\frac{[r]^{\gamma-1}}{[\rho]^{\gamma-1}}}$ i.e $T_3 = \frac{T_1 \times \rho}{[r]^{\gamma-1}} \times [\rho]^{\gamma-1}$

$\therefore T_3 = \frac{T_1\ \rho^{1+\gamma-1}}{[r]^{\gamma-1}}$ $[\rho^{\cancel{1}+\gamma-\cancel{1}} = \rho^\gamma]$

Hence $\boxed{T_3 = \frac{T_1\ \rho^\gamma}{[r]^{\gamma-1}}}$ --- (3)

From Process (4-5) Isentropic compression $PV^{\gamma} = C$

W.K.T (a) $P_4 V_4^{\gamma} = P_5 V_5^{\gamma} = PV^{\gamma} = C$

(b) $\frac{P_4}{P_5} = \left[\frac{V_5}{V_4}\right]^{\gamma}$

(c) $\frac{T_4}{T_5} = \left[\frac{V_5}{V_4}\right]^{\gamma-1}$

(d) $\frac{T_4}{T_5} = \left[\frac{P_4}{P_5}\right]^{\frac{\gamma-1}{\gamma}}$

Using eqn (c)

$$\frac{T_4}{T_5} = \left[\frac{V_5}{V_4}\right]^{\gamma-1}$$

$$\frac{T_4}{T_5} = \frac{1}{[r]^{\gamma-1}}$$

$\left[\because \frac{V_4}{V_5} = r \right.$, Compression Ratio

$\left. \therefore \frac{V_5}{V_4} = \frac{1}{r} \right]$

$$\therefore T_4 \times [r]^{\gamma-1} = T_5$$

i.e $\boxed{T_5 = T_4 \times [r]^{\gamma-1}}$ --- (4)

From Process (5-1) Constant Volume Heating $[V=C]$

Using general gas equation

$$\frac{P_5 \cancel{V_5}}{T_5} = \frac{P_1 \cancel{V_1}}{T_1} \quad [\because V_5 = V_1]$$

$$\frac{P_5}{T_5} = \frac{P_1}{T_1}$$

i.e $\frac{T_1}{T_5} = \frac{P_1}{P_5}$

Here $T_5 = T_4 \times [r]^{\gamma-1}$ eqn (4)

$$T_1 = T_5 \times \frac{P_1}{P_5}$$

$\frac{P_1}{P_5} = \alpha$ = Pressure Ratio

$\therefore \boxed{T_1 = T_4 [r]^{\gamma-1} \times \alpha}$ --- (5)

Substitute equation (5) in equation (2) and (3)

i.e Substitute value of T_1 in equation (2) and (3)

Then equation (2) becomes

$$T_2 = T_1 \times \rho$$

$$\boxed{\therefore T_2 = T_4 [r]^{r-1} \alpha \times \rho} \quad --- (6) \qquad \left[\because T_1 = T_4 [r]^{r-1} \alpha\right]$$

Then equation (3) becomes

$$T_3 = \frac{T_1 \rho^r}{[r]^{r-1}}$$

$$\therefore T_3 = \frac{T_4 \cancel{[r]^{r-1}} \alpha \times \rho^r}{\cancel{[r]^{r-1}}}$$

Hence $\boxed{T_3 = T_4 \alpha \rho^r}$ --- (7)

Now substituting T_1, T_2, T_3 and T_5 in equation (1)

i.e substituting eq'n (5), (6), (7) and (4) in eq'n (1)

$$\text{i.e} \quad \eta = 1 - \frac{(T_3 - T_4)}{r(T_2 - T_1) + (T_1 - T_5)}$$

$$\eta = 1 - \frac{(T_4 \alpha \rho^r - T_4)}{r\left(T_4 [r]^{r-1} \alpha \rho - T_4 [r]^{r-1} \alpha\right) + \left(T_4 [r]^{r-1} \alpha - T_4 [r]^{r-1}\right)}$$

$$\eta = 1 - \frac{\cancel{T_4} [\alpha \rho^r - 1]}{\cancel{T_4}[r(r^{r-1} \alpha \rho - r^{r-1} \alpha) + (r^{r-1} \alpha - r^{r-1})}$$

$$\eta = 1 - \frac{(\alpha \rho^{\gamma} - 1)}{\gamma\, r^{\gamma-1}\left[(\alpha\rho - \alpha) + (\alpha - 1)\right]}$$

$$\eta = 1 - \frac{(\alpha \rho^{\gamma} - 1)}{[r]^{\gamma-1}\, \gamma\alpha(\rho - 1) + (\alpha - 1)}$$

on rearranging above equation we get

$$\boxed{\eta = 1 - \frac{1}{[r]^{\gamma-1}}\left[\frac{(\alpha\rho^{\gamma} - 1)}{\alpha\gamma(\rho - 1) + (\alpha - 1)}\right]} \quad \text{---} (8)$$

is the efficiency of Dual combustion Cycle

Note: for Same Compression Ratio

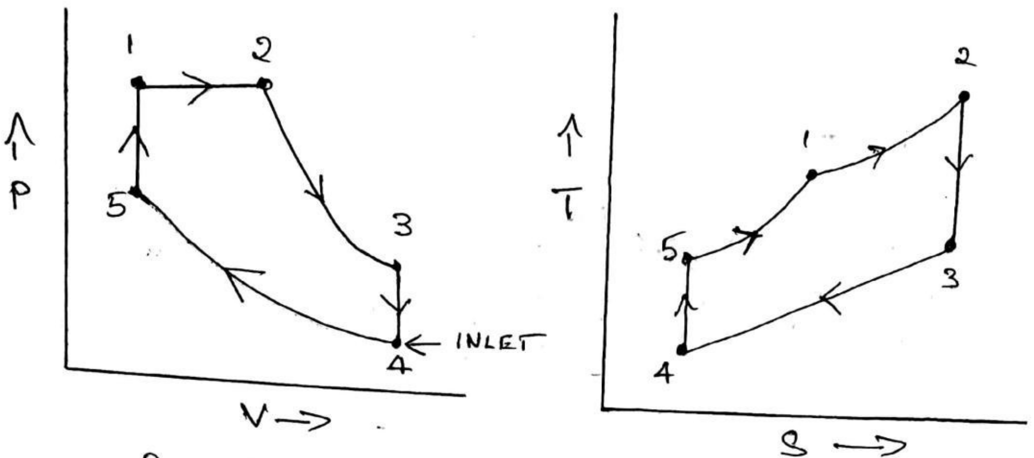
The efficiency of Dual combustion cycle is greater than Diesel cycle and less than otto Cycle

$$\boxed{\text{i.e} \quad \eta_{otto} > \eta_{Dual} > \eta_{Diesel}}$$

For same Compression Ratio (r)

Formula Used in Dual Combustion Cycle

1]



P-V diagram

T-S diagram

2] Pressure Ratio, $\alpha = \frac{P_1}{P_5}$

3] Cut off Ratio, $\rho = \frac{V_2}{V_1} = \frac{V_2}{V_5}$ $\left[\because V_1 = V_5\right]$

4] Compression Ratio, $r = \frac{V_4}{V_5}$

5] Expansion Ratio, $r_1 = \frac{V_3}{V_2} = \frac{V_4}{V_2}$ $\left[\because V_3 = V_4\right]$

6] $$\boxed{r_1 = \frac{r}{\rho}}$$

7] Heat Supplied at Constant Pressure

$$Q_{1-2} = mC_p(T_2 - T_1)$$

8] Heat supplied at Constant Volume

$$Q_{5-1} = mC_v(T_1 - T_5)$$

9] Heat Rejected at constant Volume

$$Q_{3-4} = mC_v(T_3 - T_4)$$

10] Efficiency = $\frac{\text{Heat Supplied} - \text{Heat Rejected}}{\text{Heat Supplied}}$

i.e $$\eta = \frac{[Q_{1-2} + Q_{5-1}] - [Q_{3-4}]}{[Q_{1-2} + Q_{5-1}]}$$

11] Efficiency = $\frac{\text{Work Done}}{\text{Heat Supplied}}$

$$\eta = \frac{W}{[Q_{1-2} + Q_{5-1}]} = \frac{[Q_{1-2} + Q_{5-1}] - [Q_{3-4}]}{[Q_{1-2} + Q_{5-1}]}$$

12] Work Done, W = Heat Supplied − Heat Rejected

i.e $$W = [Q_{1-2} + Q_{5-1}] - [Q_{3-4}]$$

13] Efficiency in terms of Temperature, $$\eta = 1 - \frac{(T_3 - T_4)}{\gamma(T_2 - T_1) + (T_1 - T_5)}$$

14] Efficiency, $$\eta = 1 - \frac{1}{[r]^{\gamma-1}} \times \left[\frac{(\alpha \rho^{\gamma} - 1)}{\alpha \gamma(\rho - 1) + (\alpha - 1)}\right]$$

15] $$\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$$

16] Compression ratio [r]

$$r = \frac{V_c + V_s}{V_c} = \frac{\cancel{V_c}}{\cancel{V_c}} + \frac{V_s}{V_c} = 1 + \frac{V_s}{V_c}$$

i.e $\boxed{r = \frac{V_c + V_s}{V_c}}$ or $\boxed{r = 1 + \frac{V_s}{V_c}}$

17] Area of Piston or Cylinder, $A = \frac{\pi D_c^2}{4}$ in m^2

Here D_c = Bore Diameter in m

18] Swept volume, $V_s = A \times L$ in m^3

Here L = stroke or Stroke length in m

19] Example :- If cutoff is 6% of the stroke

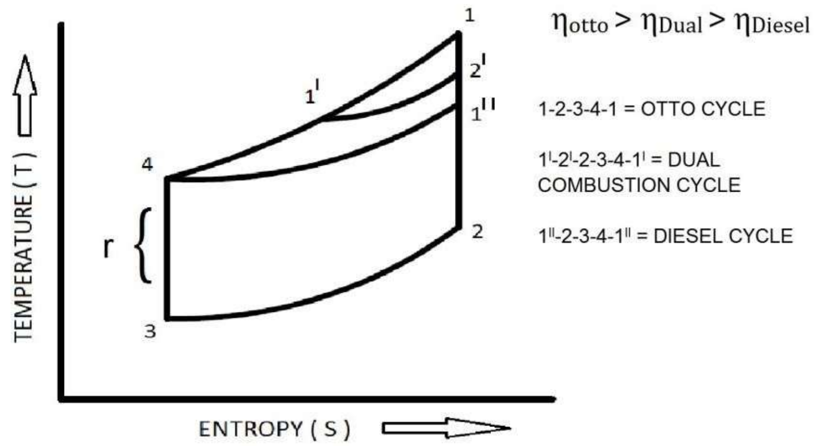
i.e $$\rho = \frac{V_c + 6\% \times V_s}{V_c}$$

$$\rho = \frac{\cancel{V_c}}{\cancel{V_c}} + \frac{\frac{6}{100} \times V_s}{V_c}$$

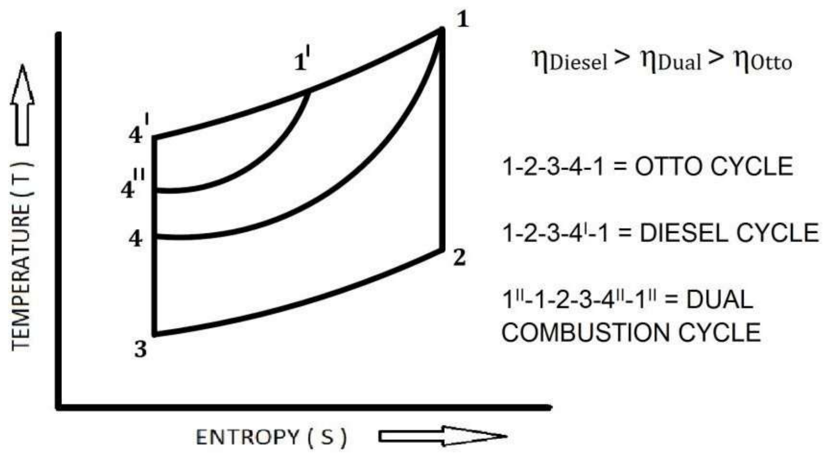
$$\boxed{\rho = 1 + 0.06 \times \frac{V_s}{V_c}}$$

Comparison between auto diesel and dual cycle

Comparison between Otto, Diesel and Dual Cycles
For same Compression Ratio (r)



Comparison between Otto, Diesel and Dual Cycles
For same Maximum Temperature (T_1) & Pressure (P_1)



Comparison Table between Otto, Diesel and Dual Cycles

SL. #	Otto Cycle	Diesel Cycle	Dual Combustion Cycle
1	It consists of two adiabatic and two constant volume process	It consist of one constant pressure process, one constant volume process and two adiabatic process	It consists of one constant pressure process, two constant volume process and two adiabatic process
2	Compression ratio is equal to expansion ratio	Compression ratio is not equal to expansion ratio	Compression ratio is not equal to expansion ratio
3	Heat Supplied at constant volume process i.e V=C	Heat Supplied at constant pressure process i.e P=C	Heat Supplied at constant volume process and constant pressure process i.e V=C and P=C
4	Efficiency depends on compression ration only	Efficiency depends on compression ratio and cut off ratio	Efficiency depends on compression ratio, cut off ratio and pressure ratio
5	For same compression ratio Air standard efficiency is high i.e $\eta_{otto} > \eta_{Dual} > \eta_{Diesel}$	For same compression ratio Air standard efficiency is less i.e $\eta_{otto} > \eta_{Dual} > \eta_{Diesel}$	For same compression ratio Air standard efficiency is between Otto and diesel cycle i.e $\eta_{otto} > \eta_{Dual} > \eta_{Diesel}$
6	For same Maximum Temperature and Pressure i.e $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$	For same Maximum Temperature and Pressure i.e $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$	For same Maximum Temperature and Pressure i.e $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$
7	Heat rejected at constant volume i.e V=C	Heat rejected at constant volume i.e V=C	Heat rejected at constant volume i.e V=C
8	Work done is done at constant entropy i.e S=C, adiabatic process	Work done is done at constant entropy i.e S=C, adiabatic process	Work done is done at constant entropy i.e S=C, adiabatic process