Heat Transfer - Introductory Concepts and Definitions: Modes of heat transfer: Basic laws governing conduction, convection, and radiation heat transfer; Thermal conductivity; convective heat transfer coefficient; radiation heat transfer; combined heat transfer mechanics. Boundary conditions of 1st, 2nd and 3rd Kind, simple problems

Introduction:

Heat transfer can be defined as the process of transformation of heat energy from one region to another region due to the temperature difference between these two regions.

Method of Heat Transfer:

1. Conduction of Heat

Heat conduction is a process in which heat is transferred from the hotter part to the colder part in a body without involving any actual movement of the molecules of the body. Heat transfer takes place from one molecule to another molecule as a result of the vibratory motion of the molecules. Heat transfer through the process of conduction occurs in substances which are in direct contact with each other. It generally takes place in solids.

Conduction example: When frying vegetables in a pan. Heat transfer takes place from flame to the pan and then to the vegetables.

Based on the conductivity of heat, substances can be classified as conductors and insulators. Substances that conduct heat easily are known as conductors and those that do not conduct heat are known as insulators.

2. Convection of Heat

In this process, heat is transferred in the liquid and gases from a region of higher temperature to a region of lower temperature. Convection heat transfer occurs partly due to the actual movement of molecules or due to the mass transfer.

For example. Heating of milk in a pan.

3. Radiation of Heat

It is the process in which heat is transferred from one body to another body without involving the molecules of the medium. Radiation heat transfer does not depend on the medium.

For example: In a microwave, the substances are heated directly without any heating medium.

Basic laws governing conduction, convection, and radiation heat transfer

The basic law governing heat conduction is Fourier's Law

This law states that the time **rate of heat transfer** through a material is **proportional to** the negative **gradient in the temperature** and to the area, at right angles to that gradient, through which the heat flows.

$$Qx \propto -\frac{dT}{dx}$$

$$Qx \propto A$$

$$Qx = K\,A\left(\frac{dT}{dx}\right)$$

K = Thermal conductivity

Q = Amount of heat flow through the body in a unit time

A = Surface area of heat flow, which is right angle to the direction of flow

dT = Temperature difference on the two faces of the body

dx = Thickness of the body through which the heat flows

K = constant of proportionality of thermal conductivity of a body

Example- Heat transfer by conduction through slab

Consider a solid slab with one of its face (say left) at higher temperature and the other (say right) at lower temperature.

Let,

T_1 = Temperature of left face in K

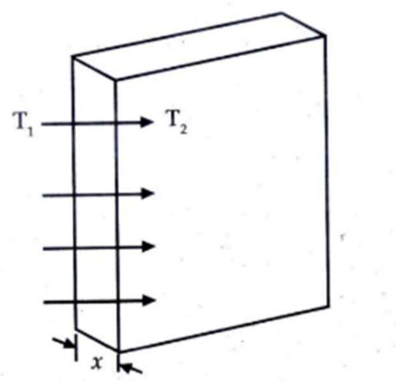
T_2 = temperature right face in K

x = Thickness of the slab

A = Area of the slab

K = Thermal conductivity of the slab

t = Time taken for heat transfer in sec



Heat transfer through a slab

w.r.t Fourier's law is given by:

$$Q = K\,A\frac{dT}{dx}$$

∴ Rate of heat flow per sec

Q = $\frac{K\,A(T_1-T_2)}{x}$

∴ Total amount of heat flow in time t is given by per sec

Q = $\frac{K\,A(T_1-T_2)t}{x}$

The basic law governing heat conduction is Newton's Law of Cooling

It states that "heat transfer from a hot body to a cold body is directly proportional to the surface area and difference of temperature between the two bodies."

i.e. Q α A (T_1-T_2)

Example Heat Transfer by conduction through composite wall:

Consider a composite wall having the tw different materials through which the heat is transferred by conduction.

Let

T_1 and T_3 = temperature of the two outer surface

T_2 = temperature at the junction

A = surface area of first material

x_1 = thickness of first material

x_2 = thickness of second material

K_1 = thermal conductivity of first materials

K_2 = thermal conductivity of second materials

T_1 is higher than T_2

∴ Heat flows from left to right

Under Steady conditions:

Rate of heat flow is section 1 = Rate of heat flow in section 2

The rate of heat flow in section 1

$$Q = \frac{K_1 A(T_1 - T_2)}{x_1}$$

or $\frac{Q x_1}{K_1 A} = T_1 - T_2$

i.e. $T_1 - T_2 = \frac{Q}{K_1} \times \frac{x_1}{A}$ ……. (1)

Rate of heat flow in section 2.

$$Q = \frac{K_2 A(T_2 - T_3)}{x_2}$$

or $\frac{Q x_2}{K_2 A} = T_2 - T_3$

i.e. $T_2 - T_3 = \frac{Q}{K_2} \times \frac{x_2}{A}$ ……. (2)

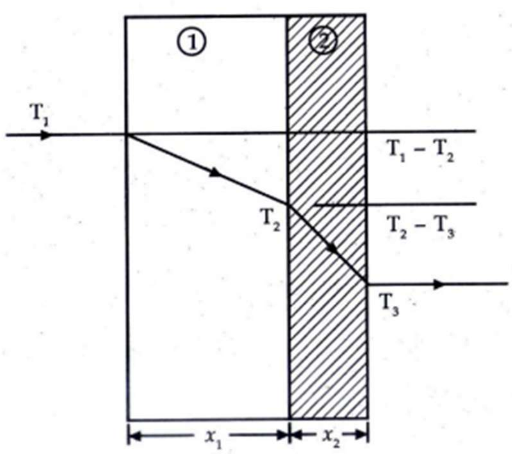
adding equation (1) and (2)

$$T_1 - T_2 + T_2 - T_3 = \frac{Q}{K_1} \times \frac{x_1}{A} + \frac{Q}{K_2} \times \frac{x_2}{A}$$

$$T_1 - T_3 = \frac{Q}{A}\left[\frac{x_1}{K_1} + \frac{x_2}{K_2}\right]$$

$$\therefore Q = \frac{A(T_1 - T_3)}{\left|\frac{x_1}{K_1} + \frac{x_2}{K_2}\right|}$$

Total heat flow in any time (t)



Heat transfer through a composite wall

The basic law governing heat conduction is Stefan –Boltzman Law of radiation

It states that "The total energy radiated per unit surface area of a black body per unit time, is directly proportional to the fourth power of the black body's thermo-dynamic temperature"

i.e. $j = \sigma T^4$,in j/m^2s

Where,

j = total energy radiated per unit surface area of a black body

σ = Stefan –Boltzman constant = 5.6704×10^{-5} , W/m^2K^4

T = absolute temperature of a black body

Example -Radial Heat Transfer by conduction through thick cylinder:

(Heat Transfer through Boiler Tubes)

Consider a thick pipe of length 'l' carrying the steam or a hot liquid at a higher temperature.

T_1 = Inside higher temperature of the liquid

T_2 = Outside lower temperature of the surroundings

r_1 = Inside radius of the pipe

r_2 = outside radius of the pipe

r_2 - r1 = thickness of the pipe

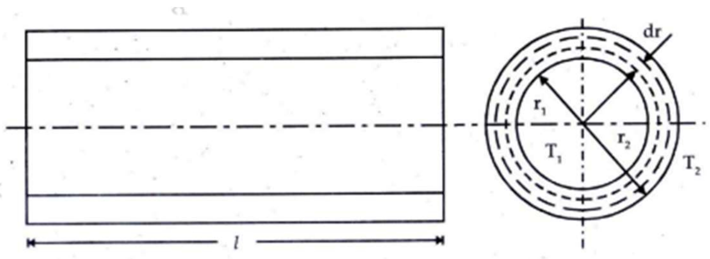
l = length of the pipe

Consider thick cylinder consists of a large number of thin concentric cylinders.

 Now consider a thin cylinder of thickness ‘dr’

 Temperature drop across the thickness be dT

 Surface Area, A = 2πrl



Heat transfer through a thick cylinder

Heat conduction

$$Q = - K A \frac{dT}{dr}$$

$$Q = -K2\, \pi r l \left[\frac{dT}{dr} \right]$$

$$\frac{dr}{r} = \frac{-2\,\pi\, l\, K\, dT}{Q}$$

Integrating above equation

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{-2\pi l K}{Q} \int_{T_1}^{T_2} dT$$

$$[\log_e r]_{r_1}^{r_2} = \frac{-2\pi l K}{Q} [T]_{T_1}^{T_2}$$

$$\log_e \left[\frac{r_2}{r_1}\right] = \frac{-2\pi l K}{Q} (T_2 - T_1)$$

$$\log_e \left[\frac{r_2}{r_1}\right] = \frac{2\pi l K}{Q} (T_1 - T_2)$$

$$2.3 \log \left[\frac{r_2}{r_1}\right] = \frac{2\pi l K}{Q} (T_1 - T_2)$$

$$Q = \frac{2\pi l K (T_1 - T_2)}{2.3 \log\left[\frac{r_2}{r_1}\right]}$$

Thermal Conductivity: “Quantity of heat in joules that flows in onesecond through one metre cube of a material when opposite faces are maintained at a temperature difference of 1K”

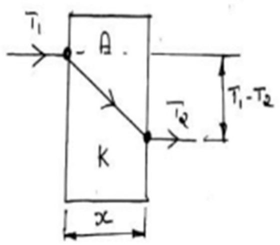
$$Q = \frac{K\, A (T_1 - T_2) t}{x}$$

Considering A = $1m^2$, T_1-T_2 = 1 K, t = 1 sec, x = 1 m

∴ Q = K

Heat flow per second, the above equation can be written as

$$Q = \frac{K\, A (T_1 - T_2)}{x}$$



Convective Heat Transfer Coefficient

The convective heat transfer coefficient, h, can be defined as: The rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference.

$$h = \frac{q}{\Delta T}$$

where

q is the local heat flux density [$W.m^{-2}$]

h is the heat transfer coefficient [$W.m^{-2}.K$]

ΔT is the temperature difference [K]

The **convective heat transfer coefficient** is dependent upon the physical properties of the fluid and the physical situation. The convective heat transfer coefficient is not a property of the fluid. It is an experimentally determined parameter whose value depends on all the variables influencing convection such as the **surface geometry**, the **nature of fluid motion**, the **properties of the fluid**, and the **bulk fluid velocity**.

Typically, the **convective heat transfer coefficient** for **laminar flow** is relatively low compared to the **convective heat transfer coefficient** for **turbulent flow**. This is due to turbulent flow having a **thinner stagnant fluid film layer** on the heat transfer surface.

Radiation Heat Transfer

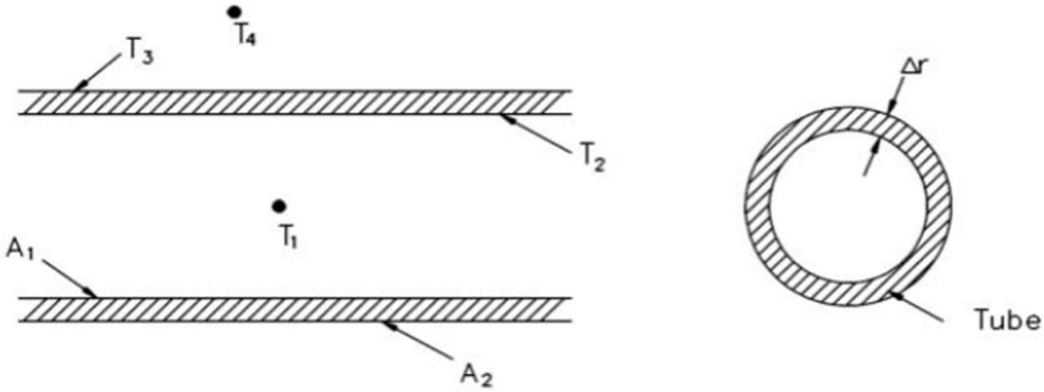
Radiation heat transfer is mediated by electromagnetic radiation, known as thermal radiation, that arises due to the temperature of a body.

Any material that has a temperature above absolute zero gives off some **radiant energy**. Most energy of this type is in the **infra-red region** of the electromagnetic spectrum although some of it is in the visible region. One of most important examples of radiation heat transfer is the Earth's absorption of solar radiation, followed by its outgoing thermal radiation. These processes determine the temperature and climate of the Earth.

(refer stefann Boltzmann law above)

combined heat transfer mechanics

Combined heat transfer mechanics is the one where conduction convection and radiation acts at the same time on the body



Three Process –

Heat Transfer by convection between temperatures T1 and T2;

Heat transfer by conduction between temperature T2 and T3

Heat transfer occurs by radiation between T3 and T4

Each has an associated heat transfer coefficient, cross- sectional area for heat transfer and temperature difference

Overall heat transfer coefficient in cylindrical geometry

$$U_o = \frac{1}{\frac{1}{h_1} + \frac{\Delta r}{k} + \frac{1}{h_2}}$$

Where,

- $\Delta \mathrm{r} = r_o - r_{in}$
- **K = Thermal conductivity coefficient of the tube wall**
- **h_1 = Convective heat transfer coefficient inside tube**
- **h_2 = Convective heat transfer coefficient outside tube**

Boundary conditions of 1st, 2nd and 3rd Kind

This type of boundary condition is also known as temperature boundary condition. Consider a plate of thickness L as shown in Fig 2-7. Let the boundary surface at x = 0 be maintained at a uniform temperature T_1 and that at x=L at uniform temperature T_2

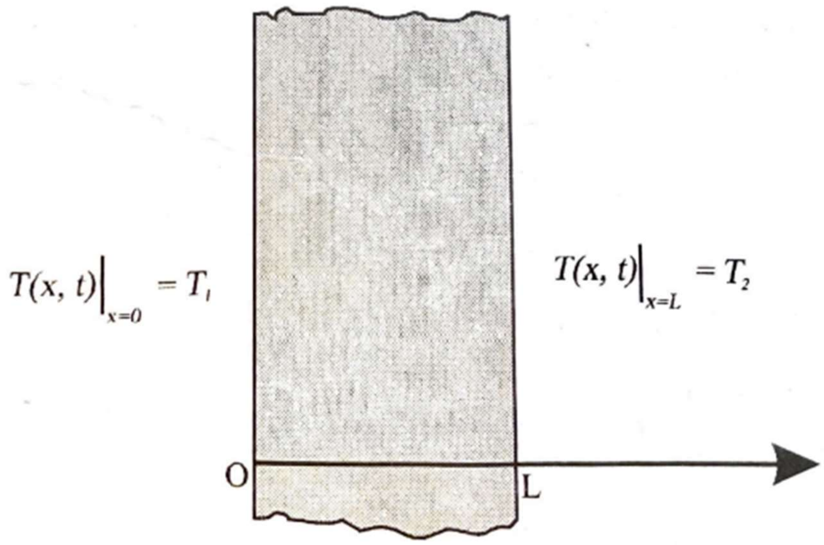


Fig 2-7 : Boundary conditions of first kind.

The Prescribed temperature boundary conditions at both surface of the plate are given as

$$T(x,t)|_{x=0} \equiv T(0,t) = T_1$$
$$T(x,t)|_{x=L} \equiv T(0,t) = T_2$$

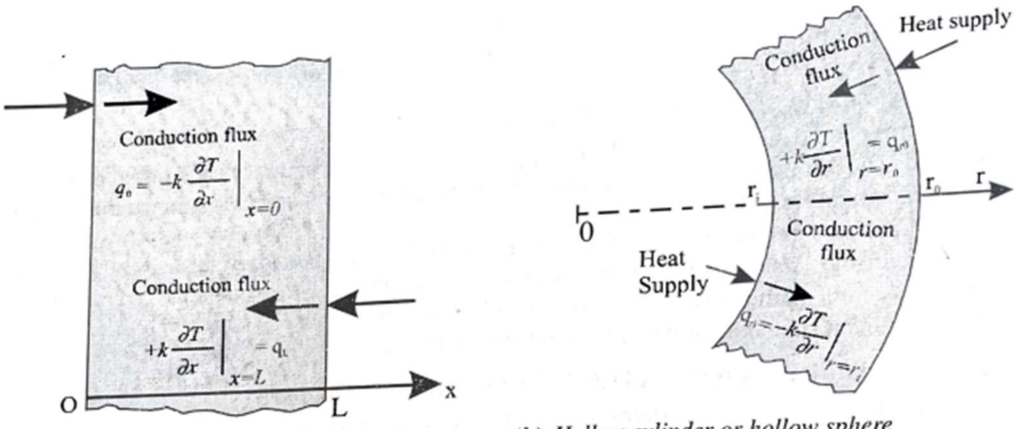
On the similar considerations boundary conditions at the surfaces of the cylinders and spheres can be determined.

When the value of temperature is prescribed at the boundary surface, the boundary condition is known as first kind.

Boundary conditions of second kind (B.C Second Kind)

This type of boundary condition is also known as heat flux boundary condition.

Consider a plate of thickness L as shown in Fig 2-8. Let g, be the rate ofheat supply through the boundary surface at x = 0 and q_L, be the rate of heat supply through the boundary surface at X =L.



(a) Plate (b) Hollow cylinder or hollow sphere

Fig 2-8 : Boundary conditions of Second kind

The prescribed heat flux boundary conditions at both the surfaces are given by equating the conduction heat flux with the external heat supply.

At the boundary surface x = 0,

$$q_0 = -k \left.\frac{\partial T}{\partial x}\right|_{x=0} \quad \text{--- [3]}$$

At the boundary surface x = L,

$$q_L = +k \left.\frac{\partial T}{\partial x}\right|_{x=L} \quad \text{--- [4]}$$

In the above two equation positive values of q_o, & q_L mean heat flow into the medium, whereas negative values of q_o, & q_L, mean heat flow from the medium.

The results similar to equations (3) and (4) are applicable to both cylindrical and spherical surfaces. Thus the conditions become

$$q_{ri} = -k \left.\frac{\partial T}{\partial r}\right|_{r=r_i} \quad \text{--- [5]}$$

and

$$q_{ro} = +k \left.\frac{\partial T}{\partial r}\right|_{r=r_0} \quad \text{--- [6]}$$

Boundary condition of Third Kind (B.C. Third Kind)

This type of boundary condition is also known as convection boundary condition.

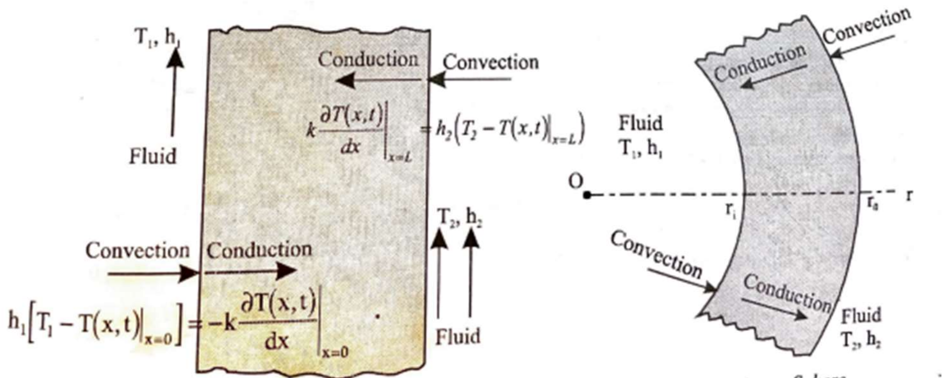
Consider a plate of thickness L as shown in Fig 2-9. Consider a fluid flowing over the surface of the plate at a temperature T_1 with a heat transfer coefficient h_1 at x = 0. For equilibrium equating the convection and conduction heat transfer, Convection heat flux into the surface = Convection heat flux from the surface

$$h_1[\,T_1 - T(x,t)|_{x=0}\,] = -k\frac{\partial T}{\partial x}(x,t)\bigg|_{x=0} \qquad \text{--- [7]}$$

At the other end of the plate when x = L let the temperature and heat transfer coefficient of the fluid be T_2 and h_2 respectively. Again for energy balance,

Convection heat flux into the surface = Conduction heat flux from the surface

$$h_2\left[\left(T_2 - T(x,t)\right)\big|_{x=L}\right] = +k\frac{\partial T}{\partial x}(x,t)\bigg|_{x=L} \qquad \text{--- [8]}$$



(a) Plate (b) Hollow cylinder or Sphere

Fig 2-9 : Boundary conditions of third kind

The results similar to equaltion [7[and [8] are applicable to both cylinders and spheres. Thus we can write the rwo boundary conditions by equating conduction and convection heat flux.

$$h_1[T_1 - T(r,t)|_{r=r_i}] = -k\frac{\partial T}{\partial r}(r,t)\bigg|_{r=r_i} \qquad \text{--- [9]}$$

$$h_2[T_2 - T(r,t)|_{r=r_0}] = k\frac{\partial T}{\partial r}(r,t)\bigg|_{r=r_0} \qquad \text{--- [10]}$$