Worked Examples

1. ***A company manufactures FM radios and calculators. The radios contribute Rs.100 per unit and calculators Rs.150 per unit as profit. Each radio requires 4 diodes and 4 resistors while each calculator requires 10 diodes and 2 resistors. A radio takes 12 minutes and calculator takes 9.6 minutes of time on the company electronic testing machine and the product manager estimates that 160 hours of test time is available. The firm has 8000 diodes and 3000 resistors in the stock. Formulate the problem as LPP.***

Solution:

i) ***Objective Function***

Let x_1, x_2 be the number of radios and calculators to be manufactured.

Profit on one unit of radio is Rs.100/-. So the profit on x_1 number of units = $100x_1$ Profit on one unit of calculator is Rs.150/-. So the profit on x_2 number of units = $150x_2$

Total profit = $100x_1 + 150x_2$

Let the total profit be represented by Z_{max}, then $Z_{max} = 100x_1 + 150x_2$, is the objective function subjected to

ii) ***Constraints***

Number of diodes required for one radio is 4, so total number of diodes for x_1 number of radios will be $4x_1$.

Number of diodes required for one calculator is 10, so total number of diodes for x_2 number of calculators will be $10x_2$.

Hence, the diodes constraint is $4x_1 + 10x_2 \leq 8000$ (as the total number of diodes cannot exceed 8000 available in the stock)

Similarly the resistors constraint is, $4x_1 + 2x_2 \leq 3000$ and the time constraint is,

$12x_1 + 9.6x_2 \leq 9{,}600$ (160 hours = 160 × 60 minutes,)

Thus, the formulation for the given problem is,

$Z_{max} = 100x_1 + 150x_2$

Subject to

$4x_1 + 10x_2 \leq 8000$

$4x_1 + 2x_2 \leq 3000$

$12x_1 + 9.6x_2 \leq 96000$

$x_1, x_2 \geq 0$ (non negative constraint)

Non negative constraint is a default constraint which indicates that x_1, x_2 cannot be negative, in other words producing negative number of radios and calculators is meaning less.

2. ***A computer company manufactures laptops and desktops that fetch profit of Rs.700/- and 500/-unit respectively. Each unit of laptop takes 4 hours of assembly time and 2 hours of testing time while each unit of desktop requires 3 hours of assembly time and 1 hour for testing. In a given month the total number of hours available for assembly is 210 hours and for inspection is 90 hours. Formulate the problem as LPP in such a way that the total profit is maximum.***

Solution:

i) Objective Function

Let x_1 be the number of laptops, x_2 be the number of desktops.

Profit on one unit of laptop is Rs.700/-. So the profit on x_1 number of units = $700x_1$

Profit on one unit of desktop is Rs.500/-. So the profit on x_2 number of units = $500x_2$

Total profit = $700x_1 + 500x_2$

Let the total profit be represented by Z_{max}, then the objective function is $Z_{max} = 700\,x_1 + 500\,x_2$ subject to,

ii) Constraints

a) Assembly time

Number of assembly hours per one laptop is 4 hence, assembly time for x_1 number of laptops is $4x_1$ and number of assembly hours per one desktop is 3 so, the assembly time for x_1 number of desktops is $3x_2$. The total assembly time is $4\,x_1 + 3\,x_2$ which should be less than 210 hours. This constraints is represented as,

$4\,x_1 + 3\,x_2 \leq 210$

b) Inspection time

Number of inspection hours per one laptop is 2 hence, inspection time for x_1 number of laptops is $2x_1$ and number of inspection hours per one desktop is 1 so, the inspection time for x_1 number of desktops is $1x_2$. The total inspection time is $2\,x_1 + 1\,x_2$ which should be less than 90 hours. This constraints is represented as,

$2x_1 + 1x_2 \leq 90$

Thus, the formulation for the given problem is,

$Z_{max} = 700\,x_1 + 500\,x_2$ subject to the following constrains

$4\,x_1 + 3\,x_2 \leq 210$ (Assembly time)

$2\,x_1 + x_2 \leq 90$ (Inspection time)

$x_1, x_2 \geq 0$ (Non-negativity constraint)

3. ***A person requires 10, 12, and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively. A dry product contains 1, 2 and 4 units of A, B and C per carton. If the liquid product sells for Rs.3/- per jar and the dry product sells for Rs.2/- per carton, how many of each should be purchased in order to minimize the cost and meet the requirements?***

Solution:

i) ***Objective Function***

Let x_1, x_2 be the number of units of liquid and dry products. The cost of liquid product is Rs. 3 per jar and of dry product is Rs. 2 per jar. Hence, the total cost is $3x_1 + 2x_2$. Since it is cost, the objective is to minimise. Hence, the objective function is

$Z_{min} = 3x_1 + 2x_2$

Subject to,

ii) ***Constraints***

a) ***Chemical 'A':*** The liquid product contains 5 units and a dry product contains 1 unit of chemical 'A'. The total availability of chemical 'A' from both the products is $5x_1 + x_2$ and the person requires 10 units of chemical 'A'. Thus, the constraint of chemical 'A' becomes , $5x_1 + x_2 \geq 10$.

b) ***Chemical 'B':*** The liquid product contains 2 units and a dry product contains 2 units of chemical 'B'. The total availability of chemical 'B' from both the products is $2x_1 + 2x_2$ and the person requires 12 units of chemical 'B'. Thus, the constraint of chemical 'B' becomes , $2x_1 + 2x_2 \geq 12$.

c) ***Chemical 'C':*** The liquid product contains 1 unit and a dry product contains 4 units of chemical 'C'. The total availability of chemical 'C' from both the products is $x_1 + 4x_2$ and the person requires 12 units of chemical 'C'. The constraint of chemical 'C' becomes , $x_1 + 4x_2 \geq 12$.

Thus, the formulation for the given problem is,

$Z_{min} = 3x_1 + 2x_2$ is the objective function subject to,

$5x_1 + x_2 \geq 10$.

$2x_1 + 2x_2 \geq 12$

$x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

(The inequality used is ≥ as it is mentioned that the person requires minimum of 10, 12 and 12 units of chemicals.)

Note: *If the objective is to minimize, (such as cost, time or distance) then the objective function will be Z_{min}. On the other hand, if the objective is to maximize, (such as profit or sales) then the objective function will be Z_{max}*

4. ***Egg contains 6 units of Vitamin A/gram and 7 units of Vitamin B/gram and costs 20 paise / gram. Milk contains 8 units of Vitamin A/gram and 12 Units of Vitamin B/gram and costs 30 paise / gram. The daily minimum requirement of Vitamin A and B are 100 units and 120 units respectively. Formulate the problem for optimum product mix.***

Solution:

i) Objective Function

Let x_1, x_2 be the number of units of egg and milk respectively. The cost of Vitamin A/gram of egg is, 20 paise and the cost of Vitamin A/gram of milk is, 30 paise. The total cost is $20x_1 + 30x_2$. Hence, the objective function is,

$Z_{min} = 20x_1 + 30x_2$

Subject to,

ii) Constraints

$6x_1 + 8x_2 \geq 100$ (as daily minimum requirement of vitamin 'A' is 100)

$7x_1 + 12x_2 \geq 120$ (as daily minimum requirement of vitamin 'B' is 120)

Thus, the formulation for the given problem is,

$Z_{min} = 20x_1 + 30x_2$

Subject to the constraints,

$6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$ and

$x_1, x_2 \geq 0.$

5. ***Old hens can be bought at Rs.50/- each but young ones cost Rs.100/- each. The old hens lay 3 eggs / week and young hens 5 eggs/week. Each egg costs Rs.2/-. A hen costs Rs.5/- per week to feed. If a person has only Rs.2,000/- to spend for hens, formulate the problem to decide how many of each kind of hen should he buy? Assume that he cannot house more than 40 hens.***

Solution:

i) Objective Function

Let x_1, x_2 be the number of old and young hens to be purchased.

Number of eggs laid by old hens = 3, number of eggs laid by young hens = 5.

Total income from the eggs

$(3x_1 + 5x_2) \times 2 = 6x_1 + 10x_2$ ... **(i)**

(Number of eggs) × selling price

Feeding cost $(x_1 + x_2) \times 5 = 5x_1 + 5x_2$ ... **(ii)**

Income – feeding cost = profit

Therefore, profit $= x_1 + 5x_2$ - is to be maximized

Thus, $Z_{max} = x_1 + 5x_2$ - is the objective function

Subject to,

ii) ***Constraints***

$50x_1 + 100x_2 \leq 2000$ (Budget constraint)

$x_1 + x_2 \leq 40$ - (Housing capacity)

$x_1, x_2 \geq 0$ - (Non negative constraint)

6. ***A farmer has to plant two kinds of trees P and Q in a land of 400 m2 area. Each P tree requires at least 25 m2 and Q tree requires at least 40 m2 of land. The annual water requirements of P tree is 30 units and of Q tree is 15 units per tree, while at most 3000 units of water is available. It is also estimated that the ratio of the number Q trees to the number of P trees should not be less than 6/19 and should not be more than 17/8. The return per tree from P is expected to be one and half times as much as from Q tree.***

Formulate the problem as a LP model.

Solution:

Let x_1, x_2 be the number of type 'P' trees and 'Q' type trees respectively.

Note:

For the convenience, the data given may be tabulated as

	Type P	Type Q	Availability
Profit	1.5	1	–
Area	25	40	400 (m^2)
Water	30	15	3000 (units)

Now, it will be easy to formulate as,

$Z_{max} = 1.5x_1 + 1x_2$ (as the profit on type P tree is 1.5 times more than on type Q tree's profit)

Subject to,

$25x_1 + 40x_2 \leq 4,400$ (Area constraint)

$30x_1 + 15\ x_2 \leq 3,300$ (Water constraint)

It is given that ratio of number of trees of type 'Q' to that of type 'P' should be in the range of 6/19 to 17/8 (not less than 6/19 but not more than 17/8)

i.e. $\frac{17}{8} \leq \frac{x_2}{x_1} \geq \frac{6}{19}$ or

$\frac{x_2}{x_1} \geq \frac{6}{19}$, $\frac{x_2}{x_1} \leq \frac{17}{18}$ and $x_1, x_2 \geq 0$

7. ***A boat manufacturer builds two types: type A and type B boats. The boats built during the months January – June go on sale in the months July – December at a profit of Rs 2,000/- per type A boat and Rs 1,500/- per type B boat. Those built during the months July – December go on sale in the months January – June at a profit of Rs 4,000/- per day A boat and Rs. 3300/- per type B boat. Each type A boat requires 5 hours in the carpentry shop and 3 hours in the finishing shop. Each type B boat requires 6 hours in the carpentry shop and 1 hour in the finishing shop. During each half year period a maximum of Rs 12,000 hours and 15,000 hours are available in the carpentry and finishing shops respectively. Sufficient material is available to build not more than 3,000 type A boats, and 3,000 type B boats a year, How many of each type 'A' of boat should be built during each half year in order to maximize the profit. Formulate as an LPP.***

Solution:

i) ***Objective Function***

Let x_{A1} = Number of type 'A' boats built during Jan-June.

x_{A2} = Number of type 'A' boats built during July-Dec.

x_{B1} = Number of type 'A' boats built during Jan-June.

x_{B2} = Number of type 'A' boats built during July-Dec.

The objective function

$Z_{max} = 2000\ x_{A1} + 4000\ x_{A2} + 1500\ x_{B1} + 3300\ x_{B2}$ subject to,

ii) ***Constraints***

$5\ x_{A1} + 6\ x_{B1} \leq 12,000$

$5\ x_{A2} + 6\ x_{B2} \leq 12,000$

$3\ x_{A1} + x_{B1} \leq 15,000$

$3\ x_{A2} + x_{B2} \leq 15,000$

$x_{A1} + x_{A2} \leq 3,000$

$x_{B1} + x_{B2} \leq 3,000$ and

$x_{A1}, x_{A2}, x_{B1}, x_{B2} \geq 0.$

8. ***A farmer has 100 acres land. He can sell all the tomatoes, lettuce or radishes he can raise. The price he can obtain is Rs.10/- per kg for tomatoes, Rs.7/- a head for lettuce and Rs.10/- per kg for radishes. The average yield per acre is 2,000 kg of tomatoes, 3000 heads of lettuce and 1,000 kg radishes. Labor required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man-days of labour are available at Rs.100/- per man day, Formulate this problem as LPP to maximize the farmer's total profit.***

Solution:

i) ***Objective Function***

Let x_1, x_2 and x_3 be the number of acres of land in which tomatoes, lettuce and radishes are grown respectively in order to maximize the profit.

So, the farmer can produce/grow

$2000x_1$ kgs of tomatoes

$3000x_2$ heads of lettuce

$1000x_3$ kgs of radishes

Hence, the total sales income of the farmer is

$2000x_1 \times 10 + 3000x_2 \times 7 + 1000x_3 \times 10$

Expenditure on the labour is $100 (5x_1 + 6x_2 + 5x_3)$

Therefore, the farmer's net profit = (total sales income) – (total expenditure)

$Z = (20{,}000x_1 + 21{,}000x_2 + 10{,}000) - (500x_1 + 600x_2 + 500x_3)$

$Z = 19{,}500x_1 + 20{,}400x_2 + 9500x_3$

Hence, the objective function is,

$Z_{max} = 19{,}500x_1 + 20{,}400x_2 + 9{,}500\, x_3$

Subject to,

ii) ***Constraints***

$x_1 + x_2 + x_3 \leq 100$ (availability of land)

$5x_1 + 6x_2 + 5x_3 \leq 400$ (availability of man days)

$x_1, x_2, x_3 \geq 0$

9. ***A toy company manufactures two types of dolls, a basic version – doll 'A' and a deluxe version – doll 'B'. Each doll of type 'B' takes twice as long to produce as one of type 'A' and the company would have time to make maximum of 2,000 dolls per day. The supply of plastic is sufficient to produce 1,500 dolls per day (Both 'A' and 'B' combined). The deluxe version requires a fancy dress of which there are only 600 per day available. If the company makes a profit of Rs.10/- and Rs.18/- per doll on doll 'A' and 'B' respectively, then how many of each doll should be produced per day in order to maximize the total profit. Formulate the problem as LPP.***

Solution:

i) ***Objective Function***

Let x_1, x_2 be the number of dolls produced per day of type A and B respectively. Let the doll A require 't' hours so that the doll B requires $2t$ hours.

Therefore, the total time to manufacture x_1 and x_2 dolls should not exceed a 2,000t hours that is $tx_1 + 2tx_2 \leq 2000t$.

The LPP is,

$Z_{max} = 10x_1 + 18x_2$

Subject to,

ii) ***Constraints***

$x_1 + 2x_2 \leq 2000$ (availability of time)

$x_1 + x_2 \leq 1,500$ (supply of plastic material)

$x_2 \leq 600$(availability of fancy dress)

$x_1, x_2 \geq 0.$

10. ***The standard weight of a special purpose brick is 5 kg and it contains two ingredients B1 and B2, B1 costs Rs.5/- per kg and B2 costs Rs.8/- per kg. Strength considerations dictate that the brick contains not more than 4 kg of B1 and a minimum of 2 kg of B2, since the demand for the product is likely to be related to the price of the brick. Formulate the above problem as a L.P model.***

Solution:

i) ***Objective Function***

Let x_1 and x_2 be the ingredients of B_1 and B_2 in the brick respectively.

Then, $Z_{min} = 5x_1 + 8x_2$ is the objective function subject to,

ii) ***Constraints***

$x_1 \leq 4$ (strength requirement)

$x_2 \geq 2$

$x_1 + x_2 = 5$ (standard weight) and

$x_1, x_2 \geq 0$

11. ***A marketing manager wishes to allocate his annual advertising budget of Rs. 20,000 in two media groups M and N. The unit cost of the message in the media 'M' is Rs. 200 and 'N' is Rs. 300. The media M is monthly magazine and not more than two insertions are desired in one issue. At least five messages should appear in the media N. The expected effective audience per unit message for Media M is 4,000 and for N is 5,000. Formulate the problem as Linear Programming problem.***

Solution:

i) ***Objective Function***

Let x_1, x_2 be the number of times of advertising in the media M and N respectively.

Then, $Z_{max} = 4000x_1 + 5000x_2$

Subject to,

ii) ***Constraints***

$200x_1 + 300x_2 \leq 20{,}000$ (Budget limitations)

$x_1 \leq 2$ (not more than 2 insertions)

$x_2 \geq 5$ (at least 5 messages to insert) and

$x_1, x_2 \geq 0$ (non – negative constraint)

12. ***Formulate a linear programming model for the problem given. The Apex television company has to decide on the number of 27- inch and 20-inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27-inch sets and 10 of 20-inch sets can be sold per month. The maximum number of work hours available is 500 per month. A 27-inch set requires 20 work hours and 20-inch set requires 10 work hours. Each 27-inch set sold produces a profit of $120 and each 20-inch produces a profit of $80. A wholesaler agreed to purchase all the television sets produced, if the numbers do not exceed the maxima indicated by market research.***

Solution:

i) ***Objective Function***

Let x_1, x_2 be the number of 27 inch and 20 inch sets respectively to be produced.

The objective function is

$Z_{max} = 120x_1 + 80x_2$

Subject to,

ii) ***Constraints***

$x_1 \leq 40$

$x_2 \leq 10$ (at most implies not more than)

$20x_1 + 10x_2 \leq 500$ (work hours limitation) and

$x_1, x_2 \geq 0$ (non – negative constraint)

13. ***The world light company produces two light fixtures requiring both metal frame parts and electrical components. The management wishes to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of $1 and each unit of product 2, upto 60 units, gives a profit of $2, any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out. Formulate a LPP model for this problem***

Solution:

i) ***Objective Function***

Let x_1, x_2 be the number of units of product 1 and product 2 respectively.

Then, $Z_{max} = 1x_1 + 2x_2$

Subject to,

ii) ***Constraints***

$1x_1 + 3x_2 \leq 200$ (Availability of frame parts)

$2x_1 + 2x_2 \leq 300$ (Availability of electrical components)

$x_2 \leq 60$ (Restriction on quantity of product 2)

and $x_1, x_2 \geq 0$

14. ***An agricultural Research institute suggested to a farmer to spread out at least 4800 kg of a special Phosphate fertilizer and not less than 7200 kg of a special nitrogen fertilizer to raise productivity of crops in his fields. There are two sources for obtaining these – mixtures A and B. Both of these are available in bags weighing 100 kg each and they cost Rs. 40 and Rs. 24 respectively. Mixture A contains phosphate and nitrogen equivalent of 20 kg and 80 kg respectively, while mixture B contains these ingredients equivalent of 50 Kg each. Write this as a linear program to determine how many bags of each type the farmer should buy in order to obtain the required fertilizer at minimum cost.***

Solution:

To formulate the given problem as LPP we need to define the objective function, constraints.

i) ***Objective Function***

Let x_1, x_2 be the number of bags of mixtures A and B respectively.

It is given that, cost per one bag of mixture A is Rs. 40. Therefore, cost per x_1 bags is $40x_1$.

Similarly, as cost per one bag of mixture B is Rs. 24, cost per x_2 bags is $24\ x_2$.

Hence, the objective function is

$Z_{min} = 40\ x_1 + 24\ x_2$

(As it is cost, the objective is to minimize it)

ii) ***Constraints***

There are two constraints namely a minimum of 4,800 Kg of phosphate and 7,200 Kg of nitrogen ingredients are required.

It is given that each bag of mixture A contains 20 Kg and each bag of mixture B contains 50 Kg of phosphate. The total Phosphate available from bag of mixture A is $20\ x_1$ and from mixture B it is $50\ x_2$.

Hence, the total availability of the phosphate from A and B is $20x_1 + 50x_2$

Thus, the phosphate requirement (constraint) can be expressed as

$20\,x_1 + 50\,x_2 \geq 4{,}800.$

Similarly, with the given data, the nitrogen requirement would be written as

$80\,x_1 + 50\,x_2 \geq 7{,}200.$

Note: *Whenever the constraint says minimum requirement it should be at least that much or greater. Hence, mathematically ≥ is used.*

Non – negativity Constraint

The decision variables, representing the number of bags of mixtures A and B, would be non – negative.

Thus, $x_1 \geq 0,\ x_2 \geq 0$

The linear programming can now be expressed as,

$Z_{min} = 40\,x_1 + 24\,x_2$

Subject to

$20\,x_1 + 50\,x_2 \geq 4{,}800$ (Phosphate requirement)

$80\,x_1 + 50\,x_2 \geq 7{,}200$ (Nitrogen requirement) and

$x_1,\ x_2 \geq 0$ (Non – negativity constraint)

15 ***A firm engaged in producing two models namely x1,x2 performs three operations – painting, assembly and testing. The relevant data are as follows.***

		Hours required for each unit		
Unit	*Sales Price*	*Assembly*	*Painting*	*Testing*
Model x_1	*Rs. 50*	*1.0*	*0.2*	*0.0*
Model x_2	*Rs. 80*	*1.5*	*0.2*	*0.1*

Total numbers of hours available each week are as under. Assembly 600, painting 100, testing 30. The firm wishes to determine its weekly products mix so as to maximize the revenue. Write up the model.

Solution:

i) ***Objective Function***

Profit on x_1 model is Rs. 50 and profit on x_2 model is Rs. 80. Hence, $50\,x_1 + 80\,x_2$ is the total profit.

Thus, $Z_{max} = 50\,x_1 + 80\,x_2$ is the objective function.

(As it is profit, the objective is to maximize it)

ii) ***Constraints***

i. Assembly time constraint

$1.0\,x_1 + 1.5\,x_2 \leq 600$

ii. Painting time constraint

$0.2\ x_1 + 0.2\ x_2 \leq 100$

iii. Testing time constraint

$0.0\ x_1 + 0.1\ x_2 \leq 30$

iv Non – negativity Constraint

As the number of models cannot be negative x_1 , $x_2 \geq 0$.

The linear programming can now be expressed as,

$Z_{max} = 50\ x_1 + 80\ x_2$

Subject to

$x_1 + 1.5\ x_2 \leq 600$

$0.2\ x_1 + 0.2\ x_2 \leq 100$

$0.0\ x_1 + 0.1\ x_2 \leq 30$ and

$x_1,\ x_2 \geq 0$

16. ***A community farming wants to find the optimal cropping pattern in the area of 25 thousand acres. The crops are:***

	Water consumption (in feet / acre)	*Profit per acre (Rs.)*
Wheat	*9*	*2000*
Maize	*6*	*1500*
Jowar	*6.5*	*1200*

He can't use more than 50% of land for wheat. Available water is 50,000 feet. At least 20% of land must be for maize. The ratio of land devoted to wheat and jowar should not be more than 3.2. Formulate the LPP.

Solution:

i) Objective Function

Let x_1, x_2, x_3 to be the number of acres of land for wheat, maize and Jowar respectively.

$Z_{max} = 2000\ x_1 + 1200\ x_2 + 1200\ x_3$

ii) Constraints

i. Availability / Consumption of water

$9\ x_1 + 6\ x_2 + 6.5\ x_3 \leq 50{,}000$

ii. Land allocation for growing wheat

Land for wheat should not be more than 50%

$x_1 \leq 12{,}500$ (50% of total land)

iii. Land allocation for growing maize

Land for maize should be at least 20%

$x_2 \geq 5{,}000$ (20% of total land)

iv. Ratio of land

Ratio of land: It is given that land devoted to wheat and jowar should not be more than 3:2

$x_1/x_2 \leq 3/2$

v. Non- negativity constraint

$x_1, x_2, x_3 \geq 0$

#17. ***A manufacturer of packing material manufacturers two types of packing tins round and flat. Major production facilities involved are cutting and joining. The cutting department can process 300 tins of round of 500 tins of flat per hour. The joining department can process 400 tins of round or 300 tins of flat per hour. If the profit contribution of round tins is Rs. 100 per tin and that of flat is Rs. 80 per tin.Formulate the problem as linear programming problem***

Solution:

Here x_1, x_2 are not 300, 500 or 400, 300 (these numbers indicate the process capability of cutting and joining departments respectively) i.e. 300 round or 500 flat tins can be processed by cutting department and 400 round or 300 flat tins can be processed by joining department per hour. x_1, x_2 are to be determined subject to these constraints.

x_1 = No. of round tins

300, 500 are the no. of round or flat tins / hour

$x_1/300 + x_2/500$ (the total time for both the tins on cutting department)

(No. of round tins/ No. of round tins / hour)

the unit of the above ratio becomes hours as No. of round tins/ No. of round tins gets cancelled.

Further, as the production capacity or time is given per hour the above ratio should be ≤ 1

i.e $x_1/300 + x_2/500 \leq 1$, cutting department constraint

Similarly the joining department constraint is $x_1/400 + x_2/300 \leq 1$

i) ***Objective Function***

Let x_1, x_2 be the number of round, flat tins respectively.

Profit from one round tin is Rs. 100, from x_1 round tins it is 100 x_1. Similarly profit from one flat can is Rs. 80, from x_2 flat tins it is 80 x_2

The data given can be tabulated as,

	Tin	
	Round (x_1)	Flat (x_2)
Processing by cutting department/hr.	300	500
Processing by joining department / hr.	400	300
Profit	100	80

Profit from one round tin is Rs. 100/- and from flat tin is Rs. 80/-. Hence,

$Z_{max} = 100\,x_1 + 80\,x_2$

ii) ***Constraints***

Since the processing times available for cutting and joining departments are not given, let us consider a unit time and thus the constraints of cutting and joining departments may be written as follows.

$x_1/300 + x_2/500 \leq 1$, cutting department constraint

$x_1/400 + x_2/300 \leq 1$, joining department constraint

Non – negative constraint

$x_1, x_2 \geq 0$

18. ***A soft drink bottling plant has two machines A and B. Though machines A and B are designed for bottling 8 – ounce and 16 – ounce respectively, Each machine can be used on both types with some loss of efficiency.***

The following data is available'

Machine	*8 – ounce bottles*	*16 – ounce bottles*
A	*100 / minute*	*40 / minute*
B	*60 / minute*	*75 / minute*

Each machine can be run 8 – hour per day, 5 days per week. Profit on each 8 – ounce bottle is Rs. 0.50 and that on 16 – ounce bottle is Rs. 0.8. Weekly production of the drink cannot exceed 3,00,000 ounces and the market can absorb 25,000 eight – ounce bottles and 7,000 sixteen – ounce bottles per week. The production planner of the bottling plant wishes to plan the production for maximization of profit. Formulate the problem as LPP.

Solution:

i) ***Objective Function***

Let x_{1A} be the number of 8-ounce bottles on machines A/week

x_{1B} be the number of 8-ounce bottles on machine B/week

x_{2A} be the number of 16-ounce bottle on machine A/week

x_{2B} be the number of 16-ounce bottles on machine B/week profit on each 8-ounce bottle is Rs. 0.50 ad on 16-ounce bottle is Rs. 0.80

Hence, the total profit is $(x_{1A} + x_{1B}) \times 0.5 + (x_{2A} + x_{2B}) \times 0.8$

Thus, the objective function is,

$$Z_{max} = (x_{1A} + x_{1B}) \times 0.5 + (x_{2A} + x_{2B}) \times 0.8$$

ii) ***Constraints***

$\frac{x_{1A}}{100} + \frac{x_{2A}}{40} \le 2400$ (machine A)

$\frac{x_{1B}}{60} + \frac{x_{2B}}{75} \le 2400$ (machine B)

(8 hrs.per day, 5 days per week = $8 \times 5 \times 60 = 2400$)

$(x_{1A} + x_{1B}) \times 8 + (x_{2A} + x_{2B}) \times 16 \le 3{,}00{,}000$ (weekly production limit)

$(x_{1A} + x_{1B}) \le 25000$ (eight ounce-bottles)

$(x_{2A} + x_{2B}) \le 7000$ (sixteen ounce-bottles)

$x_{1A}, x_{1B}, x_{2A}, x_{2B} \ge 0.$

19. ***A plant manufactures washers and dryers. The major manufacturing departments are stamping department, motor and transmission department and final assembly department. Stamping department fabricates a large number of metal plants for both washers and dryers.***

Monthly dept. capacities are as follows:

Stamping dept.	*: 10000 washers or 10000 dryers*
Motor and transmission dept.	*: 16000 washers or 7000 dryers*
Dryer assembly dept.	*: Only 5000 dryers.*
Washer assembly dept.	*: Only 9000 washers.*

Stamping dept. can produce parts for 10000 washers or 10000 dryers per month as well as for some suitable combinations. It is assumed that there is no changeover cost from washers to dryers. A similar situation exists in motor and transmission dept. but assembly lines are separate. The contribution to monthly profit is Rs. 900/- per washer and Rs. 1200/- per dryer. Determine the number of washers and dryers to be produced.

Solution:

Let x_1, x_2 be the number of washers and dryers respectively.

Objective function is, $Z_{max} = 900\,x_1 + 1200\,x_2$ (Profit)

Subject to:

$(1/10000)\ x_1 + (1/10000)\ x_2 \leq 1$ or which is equal to $x_1 + x_2 \leq 10000$
(Constraint of stamping dept.)
$(1/10000)\ x_1 + (1/7000)\ x_2 \leq 1$ or which is equal to $7x_1 + 10x_2 \leq 7000$
(Constraint of motor and transmission dept.)
$x_1 \leq 9000$ (Assembly time constraint for washer)
$x_2 \leq 5000$ (Assembly time constraint for dryer)
$x_1, x_2 \geq 0$

20. ***ABC Company owns a paint factory that produces both exterior and interior paints for wholesale distribution. The basic raw materials A and B are used to manufacture the paints. The maximum availability of A is 6 tonne /day and that B is 8 ton/day. The requirements of raw materials /tonne of interior and exterior paints are given bellow:***

Raw material	*Exterior paint*	*Interior paint*
A	*1*	*2*
B	*2*	*1*

Market survey has established that the daily demand for interior paint cannot exceed that of exterior paint by more than one ton. The survey also shows that max demand for interior paint is limited to 2 tons/day. The wholesale price/tonne is Rs.3000 for exterior and Rs.2000 for interior paint. How much interior and exterior paint the company should produced to maximize the gross income. Formulate the above data as a LPP.

Solution:

Let x_1, x_2 be the number of exterior paint and interior paint in tons.
Objective function is $Z_{max} = 3000\ x_1 + 2000\ x_2$ (Gross income)
Subject to
$x_1 + 2\ x_2 \leq 6$ (constraint of raw material 'A')
$2x_1 + x_2 \leq 8$ (constraint of raw material 'B')
$x_2 - x_1 \leq 1$
(As it is given that the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton)
i.e. The quantity difference between exterior and interior paint should not exceed 1 ton
$x_2 \leq 2$ (Max. demand for the interior paint)
$x_1, x_2 \geq$

21. ***A company has two bottling plants one located at Bangalore and the other located at Mysore. Each plant produces 3 brands of soft drinks A, B and C. Bangalore plant can produce 1500, 3000 and 2000 bottles of A, B and C in a day respectively while the capacity of Mysore plant is 1500, 1000, 5000 bottles of A, B and C per day respectively. Market survey indicates that during the month of April there will be a minimum demand of 20,000 bottles of A, 40,000 of B and 44,000 of C. The operating cost / day for Mysore plant is Rs.4000/- and for Bangalore plant is Rs.6000/-. For how many days should the plant run in April so as to minimize production cost, while still meeting the demands (only formulate).***

Solution:

Let x_1, x_2 be the number of days that Bangalore and Mysore plants are proposed to operate.

Then the objective function is

Min. $Z = 6000x_1 + 4000x_2$

Subject to the constraints

$1500x_1 + 1500x_2 \geq 20000$ (Requirement of type 'A' bottles)

$3000x_1 + 1000x_2 \geq 40000$ (Requirement of type 'B' bottles)

$2000x_1 + 5000\,x_2 \geq 44000$ (Requirement of type 'C' bottles)

and $x_1, x_2 \geq 0$

22. ***The manager of an oil refinery has to decide upon the optimal mix of two possible blending process of which the inputs and outputs per production run are as follows:***

	Input		Output	
Process	***Crude A***	***Crude B***	***Gasoline X***	***Gasoline Y***
1	***5***	***3***	***5***	***8***
2	***4***	***5***	***4***	***4***

The maximum amounts available of crude A and B are 200 and 150 units respectively. Market requirement show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profit per production run from process 1 and process 2 are Rs.3 and Rs. 4 respectively. Formulate the problem as LP model.

Solution:

Let x_1, x_2 be the number of runs of process 1 and 2 respectively

$Z_{max} = 3x_1 + 4x_2$

Subject to

$5x_1 + 4x_2 \leq 200$ (crude 'A' Constraint)

$3x_1 + 5x_2 \leq 150$ (Crude 'B' Constraint)

$5x_1+4x_2 \geq 100$ (Gasoline'X'Constraint)

$8x_1+4x_2 \geq 80$ (Gasoline'Y'Constraint)

$x_1, x_2 \geq 0$

Students are advised to declare decision variables clearly and then frame the objective function, constraints after through understanding / analysis of the problem.

23. *Solve the following LPP by graphical method.*

$Z_{max} = 3x_1 + 4x_2$

Subject to $x_1 + x_2 \leq 450$, $2x_1 + x_2 \leq 600$, $x_1, x_2 \geq 0$

Solution:

Converting the inequalities into equations we get,

$x_1 + x_2 = 450$, $2x_1 + x_2 = 600$

$x_1 + x_2 = 450$ passes through (0, 450); (450,0)

[Assuming $x_1 = 0$, $x_2 = 450$ and $x_2 = 0$, $x_1 = 450$].

Similarly, $2x_1 + x_2 = 600$ passes through, (0, 600) and (300, 0) plot the co-ordinates on the graph sheet,

Mathematically / graphically any set of values of x_1 and x_2 lying on or below the equation or line will satisfy the constraint $\leq$, in other words represent the arrows downwards if the constrain is $\leq$ and upwards if the constraint is $\geq$.

Now, the common area shared by considering all these constraints is the feasible region or the solution space (OABC) that is the most common area between upward and downward arrows in the graph.

Using the method of corner points or vertices (which are the bounded points of the region), we can find the value of Z.

$Z_{max} = 3x_1 + 4x_1$

$Z_O = 3(0) + 4(0) = 0$

$Z_A = 3(300) + 4(0) = 900$

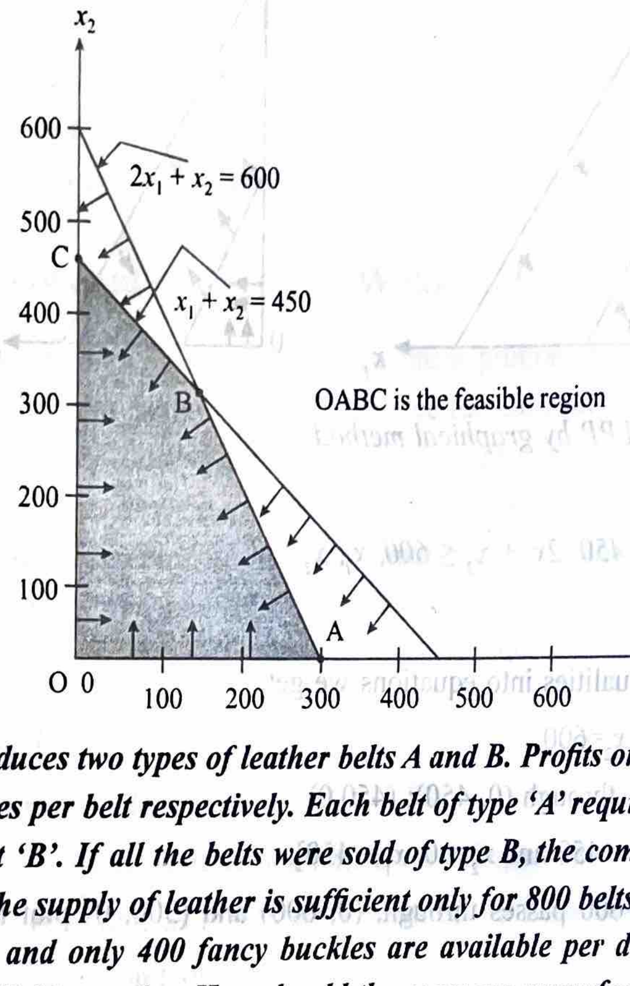
$Z_B = 3(150) + 4(300) = 450 + 1200 = 1650$

$Z_C = 3(0) + 4(450) = 1800$

Z_{max} occurs at C, the value = 1800.

Hence, the corresponding co-ordinates of x_1, x_2 are

$x_1 = 0, x_2 = 450$



24. ***A company produces two types of leather belts A and B. Profits on the two types of belts are 40 and 30 rupees per belt respectively. Each belt of type 'A' requires twice as much time as required by belt 'B'. If all the belts were sold of type B, the company could produce 1000 belts per day. The supply of leather is sufficient only for 800 belts per day. Belt 'A' requires a fancy buckle and only 400 fancy buckles are available per day. For belt 'B' only 700 buckles are available per day. How should the company manufacture the two types of belts in order to have maximum overall profit?***

Solution:

Let the company produces two types of leather belts, Type – A and Type – B profits on two types of belts are Rs. 40 and Rs. 30 respectively per bell.

i) ***Objective Function***

$Z_{max} = 40\,x_1 + 30\,x_2$

ii) ***Constraints***

Since belt of type 'A' requires twice as much time as required for a belt of type – B and the company could produce 1000 belts/days.

$2x_1 + x_2 \leq 1000$

But the supply of leather is sufficient only for 800 belts/day.

$x_1 + x_2 \leq 800$

Since, belt 'A' requires a fancy buckle and only 400 fancy buckles are available / day

$x_1 \leq 400$

Similarly, for belt of type A only 700 buckles are available / day.

$x_2 \leq 700$

Non – negative constraint: $x_1 \geq 0, x_2 \geq 0$

Thus, the formulation is

$Z_{max} = 40\, x_1 + 30\, x_2$

Subject to

$2\, x_1 + x_2 \leq 1000$ (1)

$x_1 + x_2 \leq 800$ (2)

$x_1 \leq 400$ (3)

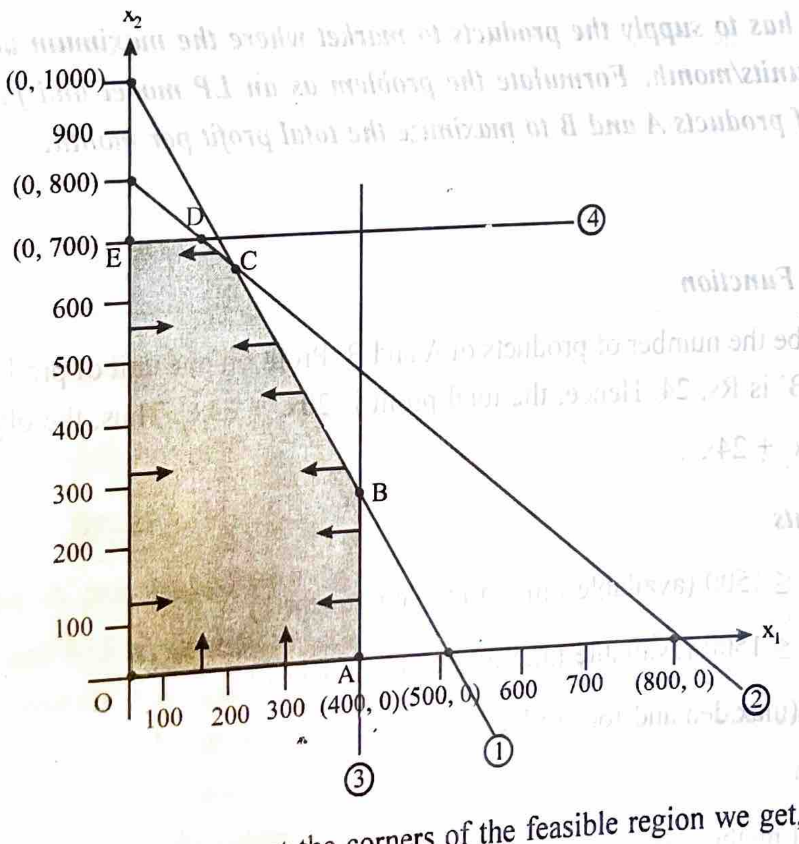
$x_2 \leq 700$ (4)

and $x_1,\ x_2 \geq 0$

Solution by Graphical Method

$2\, x_1 + x_2 = 1000$ passes through (0, 1000); (500, 0) and $x_1 + x_2 = 800$ passes through (0, 800); (800, 0); the co-ordinates of $x_1 \leq 400$ are (400, 0) and of $x_2 \leq 700$ are (0, 700)

Plotting these as straight lines on the graph sheet OABCD is the feasible region.



Substituting the values of x_1, x_2 at the corners of the feasible region we get,

$Z_0 = 40\,(0) + 30(0) = 0$

$Z_A = 40\ (400) + 30(0) = 16,000$

$Z_B = 40\ (400) + 30(200) = 22,000$

$Z_C = 40\ (200) + 30(600) = 26,000$

$Z_D = 40\ (100) + 30(700) = 25,000$

Z_{max} occurs at $x_1 = 200, x_2 = 600$

$Z_{max} = 26,000$

25. ***A plant manufactures two products A and B. The profit contribution of each product has been estimated to be Rs. 20 and Rs. 24 for products A and B respectively. Each product passes through two departments of the plant. The time required for each product and the total time available in each department are as follows:***

Department	*Time (hrs) required/unit of*		*Available time (hrs) per month*
	Product - A	*Product - B*	
1	*2*	*3*	*1500*
2	*3*	*2*	*1500*

The plant has to supply the products to market where the maximum demand for product B is 450 units/month. Formulate the problem as an LP model and find graphically, the number of products A and B to maximize the total profit per month.

Solution:

i) Objective Function

Let x_1, x_2 be the number of products of A and B. Profit on one unit of product 'A' is Rs. 20 and product 'B' is Rs. 24. Hence, the total profit is $20x_1 + 24x_2$. Thus, the objective function is,

$Z_{max} = 20x_1 + 24x_2$.

ii) Constraints

$2x_1 + 3x_2 \leq 1500$ (available time in the department 1)

$3x_1 + 2x_2 \leq 1500$ (available time in the department 2)

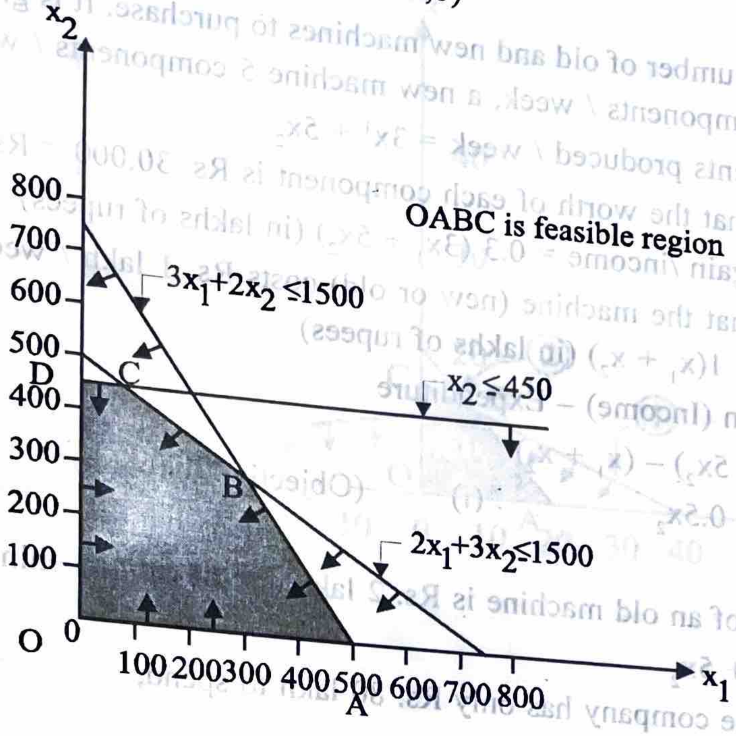
$x_2 \leq 450$ (max.demand for product 'B')

$x_1, x_2 \geq 0$.

Graphical method/solution:

$2x_1 + 3x_2 = 1500$ passes through (0,500); (750,0)

$3x_1 + 2x_2 = 1500$ passes through (0,750); (500,0)



OABCD is the feasible region. The values of the objective function at each corner of the feasible region (O, A, B, C and D are the corners of the feasible region).

$Z = 20x_1 + 24x_2$

$Z_{(O)} = 20(0) + 24(0) = 0$

$Z_{(A)} = 20(500) + 24(0) = 10,000$

$Z_{(B)} = 20(300) + 24(300) = 13,200$

$Z_{(C)} = 20(75) + 24(450) = 12,300$

$Z_{(D)} = 20(0) + 24(450) = 10,800$

Z_{max} occurs at 'B'. Hence, $x_1 = 300$, $x_2 = 300$.

Number of type 'A' product and type 'B' product are 300 each.

26 ***Old machines can be bought at Rs. 2 lakhs each and new machines at Rs. 5 lakhs each. The old machines produce 3 components / week, while new machines produce 5 components / week, each component being worth Rs. 30000. A machine (new or old) costs Rs. 1 lakh / week to maintain. The company has only Rs. 80 lakhs to spend on the machines. How many of each kind should the company buy to get a profit of more than Rs. 6 lakhs / week? Assume that the company cannot house more than 20 machines. Formulate the problem and solve it graphically.***

Solution:

Let x_1, x_2 be the number of old and new machines to purchase. It is given that an old machine can produce 3 components / week, a new machine 5 components / week.

$\therefore$ Total components produced / week = $3x^1 + 5x_2$

Also it is given that the worth of each component is Rs. 30,000 = Rs. 0.3 lakh.

Hence, the total gain /income = $0.3\ (3x_1 + 5x_2)$ (in lakhs of rupees)

It is also given that the machine (new or old) costs Rs. 1 lakh / week for maintenance. So, the expenditure = $1(x_1 + x_2)$ (in lakhs of rupees)

Profit = Total gain (Income) – Expenditure

$= 0.3\ (x_1 + 5x_2) - (x_1 + x_2)$

$Z_{max} = -0.1x_1 + 0.5x_2$ (i) (Objective function)

Constraints:

The buying cost of an old machine is Rs. 2 lakh and of new machine is Rs. 5 lakh.

Total cost = $2x_1 + 5x_2$

It is given that the company has only Rs. 80 lakh to spend,

Hence, $2x_1 + 5x_2 \leq 80$

Further it is given that the company cannot house more than 20 machines i.e. $x_1 + xz_2 \leq 20$ and the company should get a profit of more than Rs. 6 lakh / week. We already know that the profit equation as $0.1\ x_2 + 0.5\ x_2$

Thus, $-0.1\ x_1 + 0.5\ x_2 \geq 6$

The LPP is

$Z_{max} = -0.1\ x_1 + 0.5\ x_2$

$2x_1 + 5x_2 \leq 80$

$x_1 + x_2 \leq 20$

$-0.1\ x_1 + 0.5\ x_2 \geq 6$

$x_1, x_2 \geq 0$

Graphical solution:

Converting the inequalities as equations we get,

$2x_1 + 5x_2 = 80$ and it passes through (0, 16); (40, 0)

$x_1 + x_2 = 20$passes through (0, 20); (20, 0)

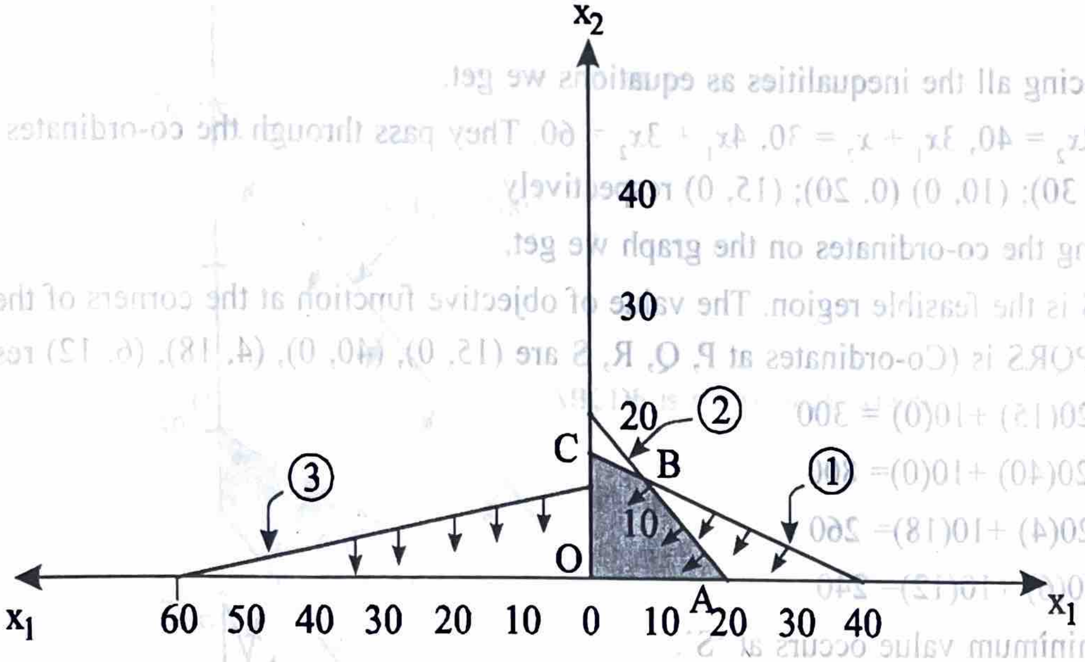
$-0.1x_1 + 0.5\ x_2 = 6$ passes through (0, 12); (– 60, 0)

Plotting the co-ordinates we get,

The feasible region is OABC

(lies in the first quadrant)

The value of the objective function at corners of the region



$Z_{(0)} = 0$

$Z_{(A)} = -0.1(20) + 0.5(0) = -2$

$$Z_{(B)} = -0.1\left(\frac{20}{3}\right) + 0.5\left(\frac{40}{3}\right) = \frac{-2}{3} + \frac{20}{3} = 6$$

$$Z_{(C)} = -0.1(0) + 0.5(16) = 8$$

Thus, the maximum profit occurs at 'C' i. e. 8 lakhs at which $x_1 = 0, x_2 = 16$

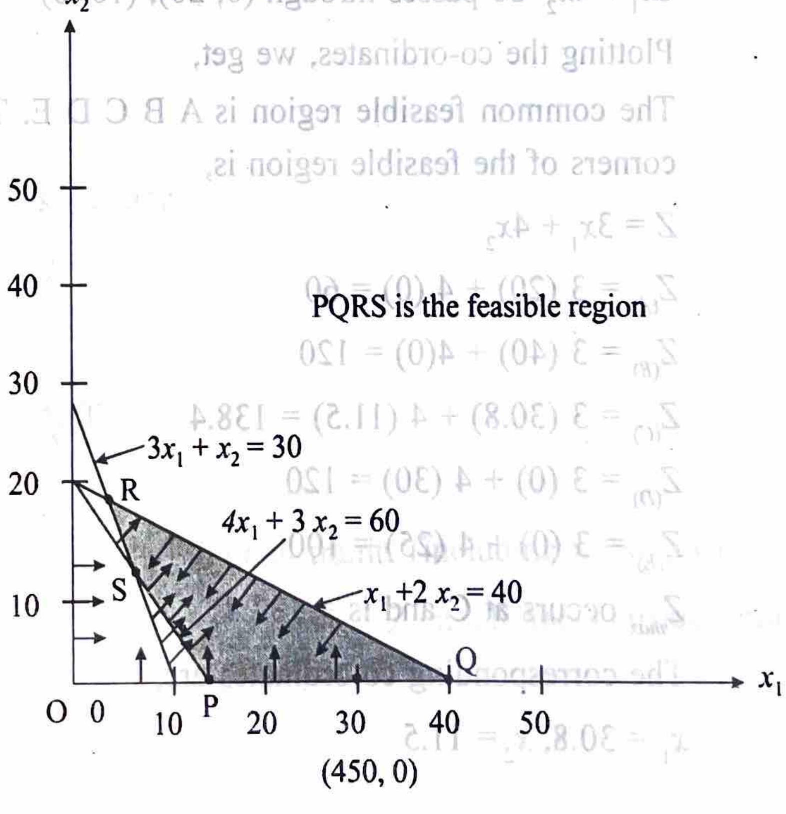
27. ***Solve the following LPP by graphical method.***

Minimize $Z = 20x_1 + 10x_2$

Subject to $x_1 + 2x_2 \leq 40$

$3x_1 + x_2 \geq 30$ $4x_1 + 3x_2 \geq 60$

$x_1, x_2 \geq 0.$



Solution:

Replacing all the inequalities as equations we get,
$x_1 + 2x_2 = 40$, $3x_1 + x_2 = 30$, $4x_1 + 3x_2 = 60$. They pass through the co-ordinates (0, 20); (40, 0) (0, 30); (10, 0) (0, 20); (15, 0) respectively.

Plotting the co-ordinates on the graph we get,

PQRS is the feasible region. The value of objective function at the corners of the feasible region PQRS is (Co-ordinates at P, Q, R, S are (15, 0), (40, 0), (4, 18), (6, 12) respectively).

$Z_P = 20(15) + 10(0) = 300$

$Z_Q = 20(40) + 10(0) = 800$

$Z_R = 20(4) + 10(18) = 260$

$Z_S = 20(6) + 10(12) = 240$

The minimum value occurs at 'S'.

Hence, $Z_{min} = 240$ for which $x_1 = 6$, $x_2 = 12$.

28. *Using graphical method find* $Z_{max} = 3x_1 + 4x_2$ *subject to*
$5x_1 + 4x_2 \leq 200$, $3x_1 + 5x_2 \leq 150$, $5x_1 + 4x_2 \geq 100$
$8x_1 + 4x_2 \geq 80$ *and* $x_1, x_2 \geq 0$

Solution:

Converting the inequalities (constraints) into equations we get,

$5x_1 + 4x_2 = 200$ passes through (0, 50), (40, 0)

$3x_1 + 5x_2 = 150$ passes through (0, 30), (50, 0)

$5x_1 + 4x_2 = 100$ passes through (0, 25), (20, 0)

$8x_1 + 4x_2 = 80$ passes through (0, 20), (10, 0)

Plotting the co-ordinates, we get,

The common feasible region is A B C D E. The value of the objective function at various corners of the feasible region is,

$Z = 3x_1 + 4x_2$

$Z_{(A)} = 3(20) + 4(0) = 60$

$Z_{(B)} = 3(40) + 4(0) = 120$

$Z_{(C)} = 3(30.8) + 4(11.5) = 138.4$

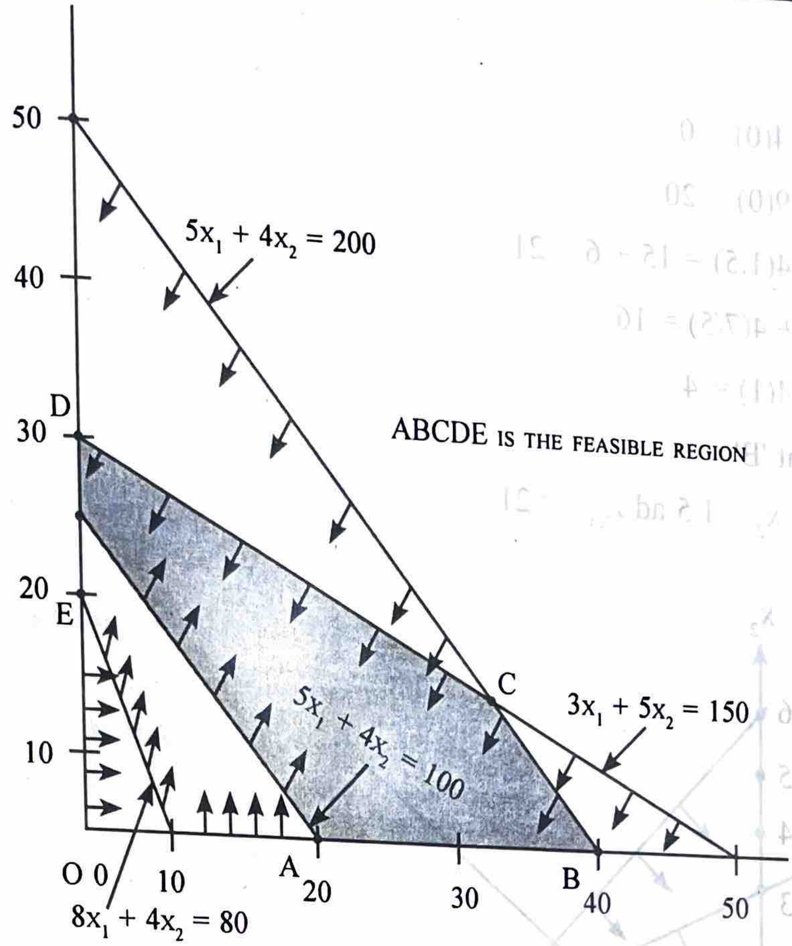
$Z_{(D)} = 3(0) + 4(30) = 120$

$Z_{(E)} = 3(0) + 4(25) = 100$

Z_{max} occurs at C and is = 138.4

The corresponding co-ordinates are,

$x_1 = 30.8$, $x_2 = 11.5$



29. $Z_{max} = 5x_1 + 4x_2$

Subject to $6x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 1$

Solution:

Converting the in-equalities as equations we get,

$x_1 + 2x_2 = 6$ passes through (0, 3) ; (6, 0)

$-x_1 + x_2 = 1$ passes through (0, 1) ; (-1, 0)

$6x_1 + 4x_2 = 24$ passes through (0, 6) ; (4, 0)

Plotting there co-ordinates

OABCD is the feasible region which satisfies all the constraints (including $x_1, x_2 \geq 0$)

By using the method of consider let us find the value of Z at all the consider points of the region i.e.

$Z_O = 5(0) + 4(0) = 0$

$Z_A = 5(4) + 9(0) = 20$

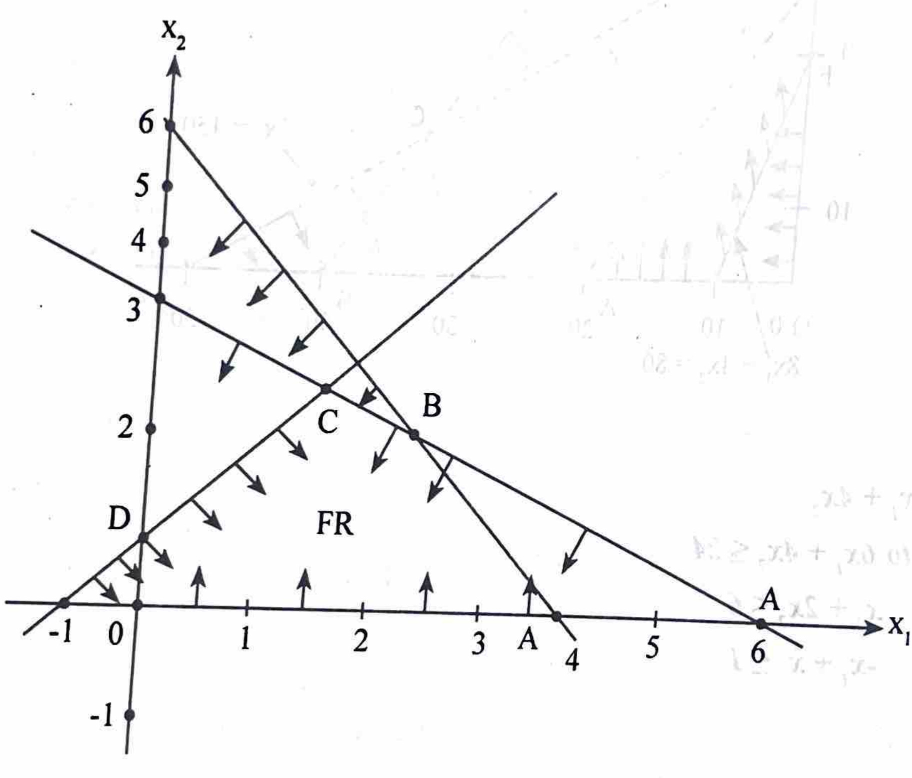
$Z_B = 5(3) + 4(1.5) = 15 + 6 = 21$

$Z_C = 5(4/3) + 4(7/5) = 16$

$Z_D = 5(0) + 4(1) = 4$

Z_{max} occurs at 'B'

Hence, $x_1 = 3$, $x_2 = 1.5$ ad $Z_{max} = 21$



30. ***Solve*** $Z_{max} = 2x_1 + 3x_2$

Subject to $x_1 + 2x_2 \leq 4$

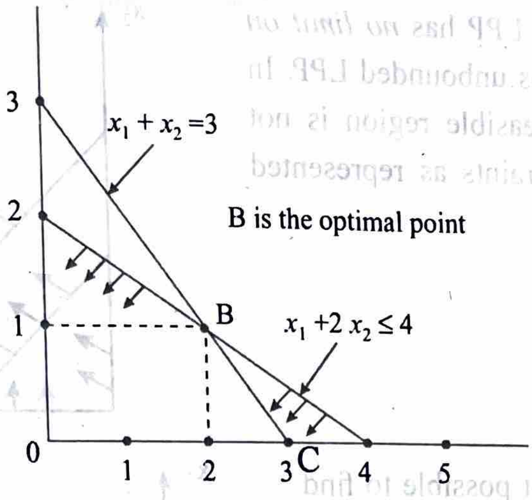
$x_1 + x_2 = 3$

$x_1, x_2 \geq 0$

$x_1 + 2x_2 = 4$ ***passes through*** *(0, 2); (4,0)*

$x_1 + x_2 = 3$ ***passes through*** *(0, 3); (3,0)*

Solution:



Plotting these coordinates on the graph sheet we get,

[As $x_1 + x_2 = 3$ is an equation, optimal point is obtained at B]

No common region is found like in previous problems.

$Z_C = 2(3) + 3(0) = 6$

B is the optimal point whose coordinates are (2, i) and $Z_{max} = 7$

Thus $x_1 = 2$, $x_2 = 1$

1.12.1 Special Cases in Graphical Method

A Linear-programming problem may be having,

a. A unique optimal solution

b. Multiple optimal solutions (alternative optimal solution)

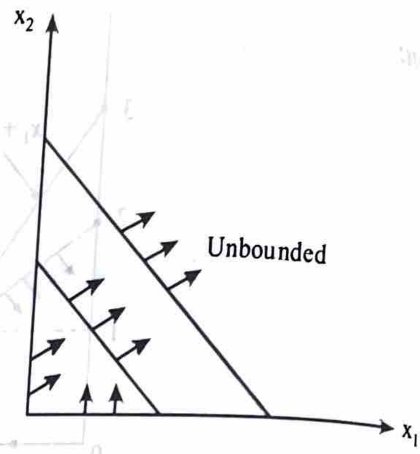
c. An unbounded solution and

d. Infeasible solution

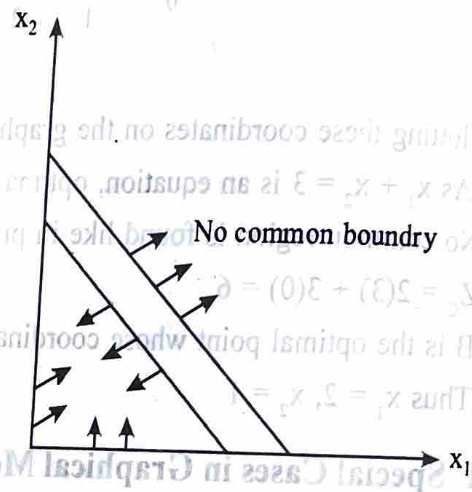
So far we have discussed linear programming problems having unique optimal solution, having one solution. The following examples will illustrate the linear programming problems having unbounded, alternate optimal solution and no solution cases.

Multiple optional solufions: In usual cases the optimal solution of any linear programming problem occurs at an extreme point of the feasible region and the solution is unique, i.e., no other solution yields the same optimum value of the objective function. However, in certain cases a given LP problem may have more than one optimal solution yielding the same objective functions value. This usually happens whenever the objective function is parallel to a constraint on alternative solution exists. e.g.:- An objective function $Z_{max} = 4x_1 + 3x_2$ with a constraint $8x_1 + 6x_2 \le 48$ will have an alternative optimal solution as the objective function is parallel to the constraint (slope of the objective function is same that of the constraint)

Unbounded Solution: If an LPP has *no limit on constraints* then it is stated as unbounded LPP. In other words, the common feasible region is not bounded by the given constraints as represented in the graph.



Infeasible Solution: If it is not possible to find a feasible solution satisfying all the constraints, then LPP is said to have an infeasible solution as represented below.



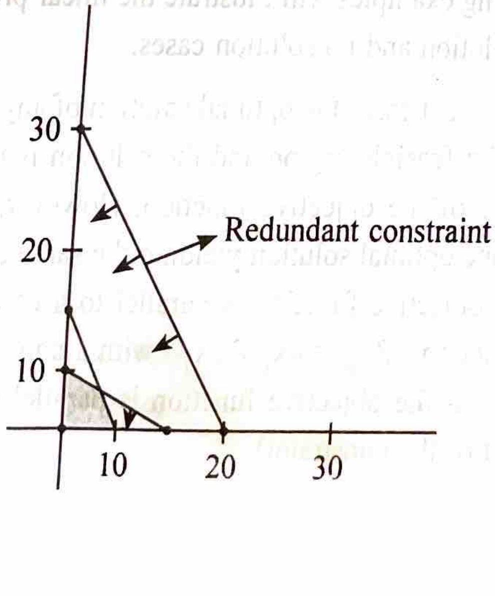
Redundant constraint: It is a system in which deletion of one of the constraints will not affect the feasible region (solution space)

Eg:

$3x_1+2x_2 \leq 30$ passes through (15, 0); (10, 0)

$3x_1+2x_2 \leq 60$ passes through (30, 0); (20, 0)

$2x_1+3x_2 \leq 30$ passes through (10, 0); (15, 0)



31. ***Solve the following LPP by graphical method***

$Z_{max} = 100x_1 + 40x_2$ ***subject to***

$5x_1 + 2x_2 \leq 1000m$

$3x_1 + 2x_2 \leq 900$

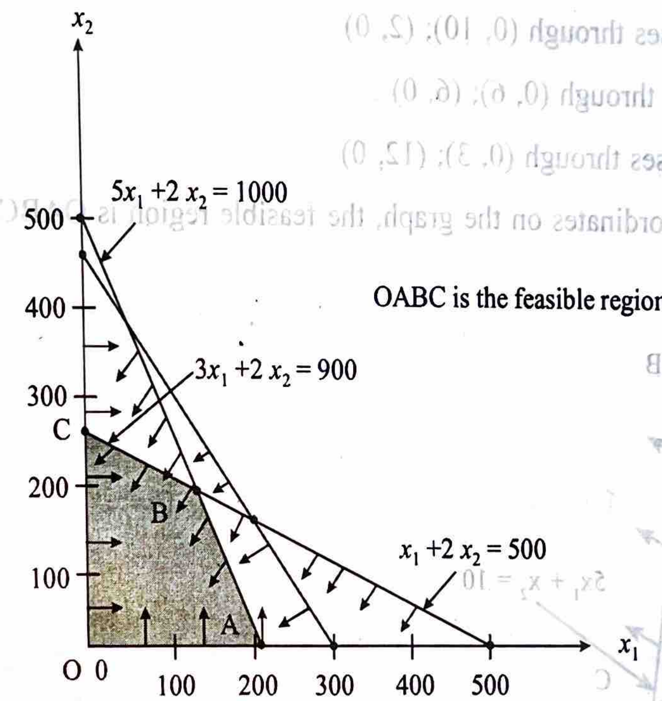
$x_1 + 2x_2 \leq 500$

$x_1, x_2 \geq 0$

Solution:

Converting the inequalities into equations we get,

$5x_1 + 2x_2 = 100$, $3x_1 + 2x_2 = 900$, $x_1 + 2x_2 = 500$, and they pass through (0, 500); (200, 0); (0, 450); (300, 0) and (0, 250); (500, 0) respectively. Plotting these co-ordinates on the graph sheet, we get,



0ABC is the feasible region. The value of objective function. $Z = 100x_1 + 40x_2$ at corner points of the feasible region.

$Z = 100x_1 + 40x_2$

$Z_0 = 100\ (0) + 4\ (0) = 0$

$Z_A = 100\ (200) + 40\ (0) = 20{,}000$

$Z_B = 100\ (125) + 40\ (187.5) = 20{,}000$

$Z_C = 100\ (0) + 40\ (250) = 10{,}000$

The maximum value of Z occurs at two vertices A and B,

That is $Z_A = Z_B = 20{,}000$

Hence, $x_1 = 200$, $x_2 = 0$ or

$x_1 = 125$, $x_2 = 187.5$ is the alternate optimal solution.

Note: *Whenever there exists more than one optimal solution the problem is said to have alternate optimal solution or infinite number of optimal solutions.*

32 ***Solve $Z_{max} = 3x_1 + 2x_2$ by using graphical method***

Subject to $5x_1 + x_2 \geq 10$

$x_1 + x_2 \geq 6$

$x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

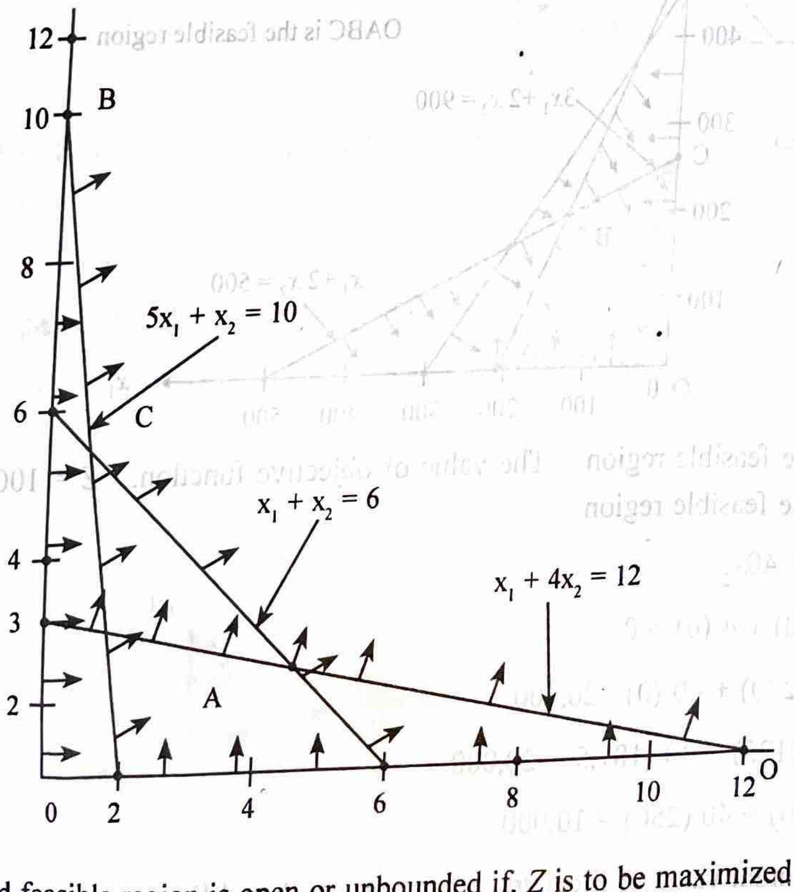
Solution:

$5x_1 + x_2 = 10$ passes through (0, 10); (2, 0)

$x_1 + x_2 = 6$ passes through (0, 6); (6, 0)

$x_1 + 4x_2 = 12$ passes through (0, 3); (12, 0)

Plotting these co-ordinates on the graph, the feasible region is OABC (open envelope).



The obtained feasible region is open or unbounded if, Z is to be maximized then the solution is unbounded that is Z_{max} occurs at infinity. Hence the solution is unbounded.

33. ***Solve*** $Z_{max} = 2x_1 + 3x_2$

Subject to $x_1 + 2x_2 \leq 4$

$x_1 + x_2 = 3$

$x_1, x_2 \geq 0$

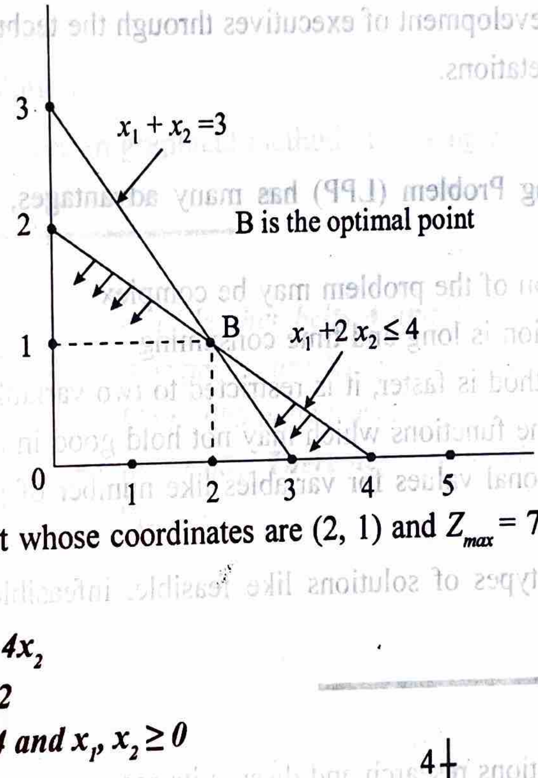
Solution:

$x_1 + 2x_2 = 4$ passes through (0, 2); (4,0)

$x_1 + x_2 = 3$ passes through (0, 3); (3,0)

Plotting these coordinates on the graph sheet we get,

[As $x_1 + x_2 = 3$ is an equation, optimal point is obtained at B]



B is the optimal point whose coordinates are (2, 1) and $Z_{max} = 7$ Thus, $x_1 = 2, x_2 = 1$.

34. ***Maximize*** $Z = 6x_1 + 4x_2$

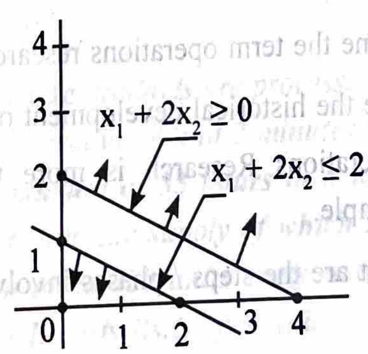
Subject to $x_1 + 2x_2 \leq 2$

$x_1 + 2x_2 \geq 4$ ***and*** $x_1, x_2 \geq 0$

Solution:

$x_1 + 2x_2 = 2$ passes through (0, 1) ; (2, 0)

$x_1 + 2x_2 = 4$ passes through (0, 2) ; (4, 0)



It is observed from the above plot that there is no common feasible region satisfying the given constraints. Hence, there is no feasible solution for the given objective function as per the constraints given.