# Operation Research

**(15ME81) – CBCS Scheme**

## Linear Programming Problem

## Module-1

By

Dr A K Murthy

[1] A Person should provide 100, 120 and 120 units of chemicals C1, C2 and C3 respectively to a Gardner. These Chemicals can be manufactured by a Liquid and a Dry product. The liquid product contains 500, 200 and 100 units of chemical C1, C2 and C3 per Jar and costs Rs 500.00 / Jar. The Dry product contains 100, 200, and 400 units of chemical C1, C2 and C3 per packet and each packet costs Rs 400.00 / packet. How many of these should produce to meet the requirement with least cost. Formulate as LPP. Jan-Feb2003

Solution [1]: $X_1$ & $X_2$ denotes number of units of Liquid and Dry products respectively.
Objective function, Minimize $Z = 500\,X_1 + 400\,X_2$
Subjected to the conditions, $500X_1 + 100X_2 \geq 100$, ( Chemical A requirements)
$200X_1 + 200X_2 \geq 120$, ( Chemical B requirements)
$100X_1 + 400X_2 \geq 120$, ( Chemical C requirements)
and $X_1$ & $X_2 \geq 0$

[2] A Firm makes 2 types of furniture chair and tables. The contribution for each product, calculated by the accounting department is Rs.200/- per chair and Rs.300/- per table. Both products are processed on 3 machines $M_1$, $M_2$, $M_3$. The time required in hours by each product and total time available in hours per week on each machine are given as follows,

| Machine | Chairs | Tables | Available time |
|---|---|---|---|
| $M_1$ | 3 | 3 | 36 |
| $M_2$ | 5 | 2 | 50 |
| $M_3$ | 2 | 6 | 60 |

Determine the product mix graphically. Dec-2010

Solution [2]: $X_C$ & $X_T$ denotes the number of chairs and tables,
Objective function, Maximize $Z = 200\,X_C + 300\,X_T$
Subjected to the conditions, $3X_C + 3X_T \leq 36$,
$5X_C + 2X_T \leq 50$,
$2X_C + 6X_T \leq 60$,
and $X_C$ & $X_T \geq 0$

[3] A manufacturer of a line of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredients available to make 20,000 bottles of A and 40,000 bottles of B, but there are only 45,000 bottles into which either of the medicines can be put. Furthermore, it takes 3 hours to prepare enough material to fill 1,000 bottles of A, it takes one hour to prepare enough material to fill 1,000 bottles of B and there are 66 hours available for this operation. The profit is Rs.80 per bottle of A and Rs.70 per bottle of B.

i) Formulate this problem as an L.P.P to maximize the profit.
ii) Solve it graphically. Jul-2006

Solution [3]: $X_1$ & $X_2$ denotes number of bottles of A and B type respectively.
Objective function, Maximize $Z = 80\,X_1 + 70\,X_2$
Subjected to the conditions , $X_1 \leq 20000$, (Bottle A limitations)
$X_2 \leq 40000$, (Bottle B limitations)
$X_1 + X_2 \leq 45000$, (Bottle Limitations)
$3X_1/1000 + 1X_2/1000 \leq 66$, (Number hours available)
and $X_1$ & $X_2 \geq 0$

[4] A plant manufactures washers and dryers. The major manufacturing departments are stamping department and final assembly department. Stamping department fabricates a large number of metal parts for both washers and dryers. Monthly dept. capacities are as follows:

Stamping dept. : 10000 washers or 10000 dryers
Motor and transmission dept. : 16000 washers or 7000 dryers
Dryer assembly dept. : only 5000 dryers, Washer assembly dept.: only 9000 washers.

Stamping dept. can produce parts for 10000 washers or 10000 dryers per month as well as for some suitable combinations. It is assumed that there is no change over cost from washers to dryers. A similar situation exists in motor and transmission dept. but assembly lines are separate. The contribution to monthly profit is Rs.900/- per washer and Rs.1200/- per dryer. Determine the number of washer and dryers to be produced. Dec-2010

Solution [4]: $X_1$ & $X_2$ denotes the number of units of washers and dryers.
Objective function, Maximize, $Z = 900\ X_1 + 1200\ X_1$.
Subjected to the conditions, $X_1/10000 + X_2/10000 \le 1$, (Time in unit month) ,
$X_1/16000 + X_2/7000 \le 1$ ,
$X_1 \le 9000$, (Capacity limitations)
$X_2 \le 5000$,
and $X_1$ & $X_2 \ge 0$

[5] Old hens can be brought at Rs 40 each and young ones at Rs 60 each. The old hens lay 15 eggs per week and young ones lay 25 eggs per week. Each egg is being worth of Rs 4.00. A hen (young or old) costs Rs 18 per week to feed. If the person has only Rs 2400/- to spend for hens, formulate the problem to decide how many of each kind of hen should he buy?, Assume that he cannot house more than 50 hens altogether and only 20 young hens available in the market. Dec-2013

Solution [5] : $X_1$ & $X_2$ be the number of old and young hens to be brought.
Revenue => $4(15\ X_1 + 25\ X_2) = 60\ X_1 + 100\ X_2$
Feeding Cost => $8(X_1 + X_2) = 8\ X_1 + 8\ X_2$
Profit = Revenue – Total Cost = $(60\ X_1 + 100\ X_2) - (8\ X_1 + 8\ X_2) = 42\ X_1 + 82\ X_2$

Objective function, Maximize, $Z = 42\ X_1 + 82\ X_2$
Subjected to the conditions $40X_1 + 60X_2 \le 2400$, (One time investment) ,
$X_1 + X_2 \le 50$ , (Housing capacity limitations)
$X_1 \le 20$, (Availability limitations)
and $X_1$ & $X_2 \ge 0$

[6] A student has two final exams to prepare for. Each hour of study he devotes to course A is expected to return him Rs.600/- in terms of long range benefits and each hour of study he devotes to course B is expected to return him Rs.300/- in terms of long range benefits. The shops are all closed; the student has only 15 chocolates, remaining. He feels he needs one chocolate every 20 minute while studying for course B and 1 every 12 minutes while studying for course A. time is running short and only 4 hours remain to prepare for exam. It is necessary that the student must devote at least 2 hours for studying. The student would like to maximize his returns for the effort expended. Solve this problem as an L.P.P. and determine the optimum policy for the student by solving the problem. Dec08-Jan09 & Jan/Feb2004

Solution [6]: $X_A$ & $X_B$ denotes the number of hours of spent to study course A and B.
Objective function, Maximize $Z = 600\ X_A + 300\ X_B$
Subjected to the conditions , $5X_A + 3X_B \le 15$, (5 chocolates per hour for A and 3 per hour for B)
$X_A + X_B \le 4$, (4 hours remain)
$X_A + X_B \ge 2$, (Minimum study of 2 hours)
and $X_A$ & $X_B \ge 0$

[7] Formulate a linear programming model for the problem given below. The apex television company need to decide on the number of 27-inch and 20-inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27-inch sets and 10 of 20-inch sets can be sold per month. The maximum number of work hours available is 500 per month. A 27-inch set requires 20 work hours and 20-inch set requires 10 work hours. Each 27-inch set sold produces a profit of $12000 and each 20-inch produces a profit of $8000. A wholesaler agreed to purchase all the television sets produced if the numbers do not exceed the maxima indicated by market research. Dec09/Jan10

Solution [7]: $X_1$ & $X_2$ denotes the number of number of 27-inch and 20-inch sets produced respectively.

Objective function, Maximize $Z = 12000X_1 + 8000X_2$

Subjected to the conditions, $X_1 \le 40$, (27-inch type can be sold)

$X_2 \le 10$, (20-inch type can be sold)

$20X_1 + 10X_2 \le 500$, ( Number of work hours available)

and $X_1$ & $X2 \ge 0$

[8] A firm plans to purchase at least 200 quintals of scrap containing high quality (x) and low quality (y) metals. It decides that scrap purchased must contain at least 100 quintals of x-metal and not more than 35 quintals of y-metal. The firm can purchase metals from two supplier's A and B in unlimited quantities. The purchase of x and y in terms of weight of scrap supplied by A and B are given below:

| Metals | Supplier A | Supplier B |
|---|---|---|
| High quality (X) | 25% | 75% |
| Low quality (Y) | 10% | 20% |

The price of scrap supplied by A and B is Rs.200/quintal and Rs.400/quintal. Formulate the problem as LP model and solve graphically to determine the quantities purchased from A and B suppliers. (2002 SCH) Dec09/Jan10

Solution [8]: $X_A$ & $X_B$ denotes quantity (in Quintals) of scrap material purchased from Supplier A and B,

Objective function, Minimize $Z = 200X_A + 400X_B$

Subjected to the conditions, $X_A + X_B \ge 200$, (Minimum purchase of scrap)

$0.25X_A + 0.75X_B \ge 100$, (Must contain x-metal type)

$0.10X_A + 0.20X_B \le 35$, (Not more than y-metal type)

and $X_A$ & $X_B \ge 0$

[9] A former need to plant two kinds of trees P and Q in a land of 4000 sq. m. area. Each P tree requires at least 25 sq. m and Q tree requires at least 40 sq. m of land. The annual water requirements P tree is 30 units and of Q tree is 15 units per tree, while at most 3000 units of water is available. It is also estimated that the ratio of the number Q trees to the number of P trees should not be less than 6/19 and should not be more than 17/8. The return per tree from P is expected to be one and half times as much as from Q tree. Formulate the problem as a LP model. May-Jun2010

Solution [9]: $X_P$ & $X_Q$ denotes number P and Q type of plants to be planted,

Return per tree from P is expected to be one and half times as much as from Q tree, So

Objective function, Maximize $Z = 1.5 X_P + X_Q$

Subjected to the conditions, $25X_P + 40X_Q \le 4000$, (Available land)

$30X_P + 15X_Q \le 3000$, (Available water)

$X_Q / X_P \ge 6/19$, (Q to P ratio Not less than)

$X_Q / X_P \le 17/8$, (Q to P ratio Not more than)

and $X_P$ & $X_Q \ge 0$

[10] The manager of an oil refinery must decide on the optimal mix of 2 possible blending process of which the input for production run as follows.

| Process | Inputs (units) | | Output (units) | |
|---|---|---|---|---|
| | Crude A | Crude B | Gasolene-X | Gasolene-Y |
| 1 | 5 | 3 | 5 | 8 |
| 2 | 4 | 5 | 4 | 4 |

The maximum amount of crude available is 200 units of crude A and 150 units of crude B. the market requirement shows that at least 100 units of gasoline X and units of gasoline Y must be produced. The profit form production run of process 1 and 2 are Rs.300 and Rs.400. formulate a suitable mathematical model and solve the same by using graphical method. Dec06-Jan07

Solution [10]: $X_1$ & $X_2$ denotes number of production runs from Process-1 and Process-2.

Objective function, Maximize $Z = 300 X_1 + 400 X_2$

Subjected to the conditions , $5X_1 + 4X_2 \leq 200$, (Crude A limitations)

$3X_1 + 5X_2 \leq 150$, (Crude B limitations)

$5X_1 + 4X_2 \geq 100$, (Gas -X Requirements)

$8X_1 + 4X_2 \geq 100$, (Gas -Y Requirements)

and $X_1$ & $X_2 \geq 0$

[11] ABC food's company is developing a calorie high protein diet supplement called HI-Pro. The specification along with calorie, protein and vitamin content of three basic foods are given in the following table.

| Nutritional elements | Basic foods | | | Hi-Pro. Specifications |
|---|---|---|---|---|
| | No. 1 | No. 2 | No. 3 | |
| Calories | 350 | 250 | 200 | At most 300 |
| Protein | 250 | 300 | 150 | At least 200 |
| Vitamin - A | 100 | 150 | 75 | At least 100 |
| Vitamin – B | 75 | 125 | 150 | At least 100 |
| Cost of serving (Rs) | 1.5 | 2.0 | 1.2 | |
| Revenue from serving(Rs) | 5.5 | 7.0 | 4.2 | |

Formulate the LPP model.

Solution [11]: Solution: $X_1$ & $X_2$ & $X_3$ denotes the number of units of basic food number 1, 2 and 3.

Profit = (Revenue - cost)

Objective function: Maximize $Z = 4X_1 + 5X_2 + 3X_3$.

Subjected to the conditions $350 X_1 + 250 X_2 + 200 X_3 \leq 300$,

$250 X_1 + 300 X_2 + 150 X_3 \geq 200$,

$100 X_1 + 150 X_2 + 75 X_3 \geq 100$,

$75 X_1 + 125 X_2 + 150 X_3 \geq 100$

and $X_1, X_2$ & $X_3 \geq 0$

[12] The following table gives the data for a problem. Formulate the problem as a LP model.
Jun-July2009 & Jun2012

| Raw Materials | Requirement/Unit | | | Availability |
|---|---|---|---|---|
| | I | II | III | |
| A | 2 | 3 | 5 | 4000 |
| B | 4 | 2 | 7 | 6000 |
| Min Demand | 200 | 200 | 150 | |
| Profit/ Unit | 30 | 20 | 50 | |

Solution [12]: $X_1$, $X_2$ & $X_3$ denotes the number of required units of I, II and III.

Objective function, Maximize $Z = 30X_1 + 20X_2 + 50X_3$

Subjected to the conditions, $2X_1 + 3X_2 + 5X_3 \leq 4000$, ( Raw material A available)
$4X_1 + 2X_2 + 7X_3 \leq 6000$, ( Raw material B available)
$X_1 \geq 200$, ( Minimum requirements of I)
$X_2 \geq 200$, ( Minimum requirements of II)
$X_3 \geq 150$, ( Minimum requirements of III)
and $X_1, X_2$ & $X_3 \geq 0$

[13] The diet for a broiler to be formulated. The required daily batch of the feed is 100kgs. The diet must contain (i) At least 0.8% but not more than 1.2% calcium. (ii) At least 22% protein. (iii) At least 5% crude fiber. The main ingredients used include limestone, Corn and Soyabean. The nutritive content of these ingredients is summarized below Formulate the model to minimize the cost. Dec-2010

| Ingredients | Calcium | Protein | Fiber | Cost/kg in Rs |
|---|---|---|---|---|
| Limestone | 0.3 | 0.0 | 0.00 | 1.65 |
| Corn | 0.001 | 0.09 | 0.02 | 4.65 |
| Soyabean | 0.002 | 0.5 | 0.08 | 12.5 |

Solution [13]: $X_L$, $X_C$ & $X_S$ denotes the quantity (in Kgs) of Lime stone, Corm and Soya bean respectively,

Objective function, Minimize, $Z = 1.65 X_L + 4.65 X_C + 12.5 X_S$

Subjected to the conditions, $(X_L + X_C + X_S) \geq 100$
$(0.3 X_L + 0.001X_C + 0.002X_S) \geq 0.008$ (at least Calcium)
$(0.3 X_L + 0.001X_C + 0.002X_S) \leq 0.012$ (at most Calcium)
$(0.0 X_L + 0.09X_C + 0.5X_S) \geq 0.22$ (at least Protein)
$(0.0 X_L + 0.02X_C + 0.08X_S) \geq 0.05$ (at least Fiber)
and $X_L$, $X_C$ & $X_S \geq 0$ (Non negativity constraints)

[14] A boat manufacturer builds two types: type A and type B boats. The boats built during the months January-June go on sale in the months July-December at a profit of Rs.2000/- per type A and Rs.1500/-per type B. Those built during the months July-December go on sale in the months January-June at a profit of Rs.4000/-per type A and Rs.3300/-per type B. Each type A required 5 hours in the carpentry shop and 3 hours in the finishing shop. Each type B required 6 hours in the carpentry shop and 1 hour in the finishing shop. During each half year period, a maximum of 12000 hours and 15000 hours are available in the carpentry and finishing shops respectively. Sufficient material is available to built no more than 3000 type A and 3000 type B boats a year. How many of each type of boat should be built during each half year in-order to maximize the profit. Formulate as the LPP.

June-July09

Solution [14]:
$X_{AD}$ & $X_{BD}$ denotes number of boats of A and B types sold during July-December and
$X_{AJ}$ & $X_{BJ}$ denotes number of boats of A and B types sold during January-June.

Objective function, Maximize $Z = 2000\,X_{AD} + 1500\,X_{BD} + 4000\,X_{AJ} + 3300\,X_{BJ}$

Subjected to the conditions , $5\,X_{AD} + 6\,X_{BD} \leq 12000$, (Available carpentry period January-June)

$5\,X_{AJ} + 6\,X_{BJ} \leq 12000$, (Available carpentry period July-December)

$3\,X_{AD} + 1X_{BD} \leq 15000$, (Available finishing period January-June)

$3\,X_{AJ} + 1\,X_{BJ} \leq 15000$, (Available finishing period July-December)

$X_{AD} + X_{AJ} \leq 3000$, (Material available)

$X_{BD} + X_{BJ} \leq 3000$, (Material available)

and $X_{AD}$ , $X_{AJ,}$ $X_{BD}$ & $X_{BJ} \geq 0$

[15] METRO Sports wishes to determine how many advertisements to place in the selected three-monthly Magazines A, B and C. Objective is to advertise in such a way that total exposure to principle buyers of expensive sports goods is maximized. Percentages of readers for each magazine are known. Exposure in any magazine is the number of advertisements placed multiplied by the number of principle buyers. The following date may be used.

| Exposure category | Magazines A | Magazines B | Magazines C |
|---|---|---|---|
| Readers (in Lakh's) | 1 | 0.6 | 0.4 |
| Principal buyers | 10% | 15% | 7% |
| Cost per advertisement (Rs) | 5000 | 4500 | 4250 |

The budgeted amount is at most Rs 1 Lakh for the advertisements. The owner has already decided that magazine 'A' should have no more than 6 advertisements and that 'B' and 'C' each have at least two advertisements. Formulate LPP model for the problem.

Solution [15]: $X_1$, X2 and X3 be the number of advertisement insertions in magazine A, B and C respectively.

Objective function: (Find the total Exposure),

Maximize Z = (10% of 100000 $X_1$+ 15% of 60000 $X_2$+7% of 40000 $X_3$)

Subjected to the conditions,

$5000\,X_1+4500\,X_2+4250\,X_3 \leq 100000$, (Budget con)

$X_1 \leq 6$,

$X_2 \geq 2$ ,

$X_3 \geq 2$ (Advertisement con)

and $X_1$ $X_2$ & $X_3 \geq 0$

[16] A city hospital has the following daily requirements for nurses.

| Period | Clock time (24 hrs a day) | Minimum number of NURSES required |
|---|---|---|
| 1 | 6 AM – 10 AM | 2 |
| 2 | 10 AM – 2 PM | 7 |
| 3 | 2 PM – 6 PM | 15 |
| 4 | 6 PM – 10 PM | 8 |
| 5 | 10 - PM – 2 AM | 20 |
| 6 | 2 AM – 6 AM | 6 |

Nurses report to the hospital at the beginning of each period and work for 8 consecutive hours. The hospital wants to determine the minimum number of nurses to be employed so that there will be sufficient number nurses available for each period. Formulate this as a linear programming problem by setting up appropriate constraints and objective function.

Solution [16]:

Let $X_1$, $X_2$, $X_3$, $X_4$, $X_5$ and $X_6$ be the number of nurses on duty at 6 AM, 10 AM, 2 PM, 6 PM, 10 PM, 2 AM and 6 AM respectively.

Objective function: Minimize Z = x1+x2+x3+x4+x5+x6

Subjected to the constraints

$X_1 + X_2 \geq 7$ ,
$X_2 + X_3 \geq 15$,
$X_3 + X_4 \geq 8$,
$X_4 + X_5 \geq 20$,
$X_5 + X_6 \geq 6$
$X_6 + X_1 \geq 2$
and $X_1, X_2, X_3, X_4, X_5$ and $X_6 \geq 0$

[17] A manufacturer of biscuits is considering three types of gift packs containing three types of biscuits; Orange (O), Chocolate (C) and Wafers (W) Cream. Market research conducted to access the preferences of consumer shows the following assortments to be in good demand.

| Assortments | Contents | Selling price per kg |
|---|---|---|
| A | Not less than 40% of O, Not more than 20% of C and any | 220 |
| B | Not less than 50% of O, Not more than 30% of C | 200 |
| C | No restrictions | 120 |

Biscuits Plant capacity and Cost of manufacturing is given below.

| Biscuit Varity | : | Orange | Chocolate | Wafers |
|---|---|---|---|---|
| Plant capacity (kg/day) | : | 200 | 200 | 150 |
| Manufacturing Cost (Rs/kg) | : | 80 | 90 | 70 |

Formulate as LP Model to maximize the profit assuming there are no market restrictions.

Solution [17]:

Assortment 'A'- Consider $X_{1a}$, $X_{2a}$ and $X_{3a}$ denote quantity in kg. of O, C & W biscuits respectively.
Assortment 'B'- Consider $X_{1b}$ and $X_{2b}$ denote quantity in kg. of O& C biscuits respectively.
Assortment 'C'- Consider $X_{1c}$, $X_{2c}$ and $X_{3c}$ denote quantity in kg. of O, C & W biscuits respectively.

Profit = Revenue - Total cost

Profit = $[220(X_{1a} + X_{2a} + X_{3a}) + 200(X_{1b} + X_{2b}) + 120(X_{1c} + X_{2c} + X_{3c})] -$
$[80(X_{1a} + X_{1b} + X_{1c}) + 90(X_{2a} + X_{2b} + X_{2c}) + 70(X_{3a} + X_{3c})]$

Objective function:

Maximize $Z = 140 X_{1a} + 120 X_{1b} + 40 X_{1c} + 130 X_{2a} + 110 X_{2b} + 30 X_{2c} + 150 X_{3a} + 50 X_{3c}$

Subjected to the constraints,

Gift pack -A- Assortment $X_{1a} \geq 0.4 (X_{1a} + X_{2a} + X_{3a})$, $X_{2a} \leq 0.2 (X_{1a} + X_{2a} + X_{3a})$,
Gift pack -B- Assortment $X_{1b} \geq 0.5 (X_{1b} + X_{2b})$, $X_{2b} \leq 0.3 (X_{1b} + X_{2b})$,
Orange constraints - $(X_{1a} + X_{1b} + X_{1c}) \leq 200$
Chocolate constraints - $(X_{2a} + X_{2b} + X_{2c}) \leq 200$
Wafers constraints - $(X_{3a} + X_{3c}) \leq 150$ Also $X_{1a}, X_{2a}, X_{3a}, X_{1b}, X_{2b}, X_{1c}, X_{2c}$ and $X_{3c} \geq 0$

[18] An oil refinery wishes to blend 3 petroleum constitutes to make 2 grades of petrol A and B. The availability and costs of the 3 constituents are given below:

| Constituents | Max. Available Barrels/Day | Costs Rs/Barrel |
|---|---|---|
| 1 | 3500 | 3000 |
| 2 | 2000 | 6000 |
| 3 | 3000 | 4000 |

To maintain the required quality of each grade of petrol, the following specifications are given along with the selling price each grade.

| Grade | Specification | Selling price(Rs/Barrel) |
|---|---|---|
| A | Not more than 30% of 1 and Not more than 50% of 3 | 5000 |
| B | Not more than 50% of 1 and Not more than 10% of 2 | 4500 |

Setup a linear programming model for determining the amount of blend in each grade of petrol. Only formulate.
May 2007 (10 marks)

Solution [18]:

$X_{1A}$ & $X_{3A}$ denotes the quantity of Constituents 1 and 3 respectively in A.

$X_{1B}$ & $X_{2B}$ denotes the quantity of Constituents 1 and 2 respectively in B.

Profit = Revenue – Cost

Revenue $= 5000 (X_{1A} + X_{3A}) + 4500 (X_{1B} + X_{2B})$ (Revenue from A and B)

Cost $= 3000 (X_{1A} + X_{1B}) + 6000 (X_{2B}) + 4000 (X_{3A})$ (Cost constituents 1,2 and 3 in A and B)

Profit $= 2000 X_{1A} + 1500 X_{1B} + 1000 X_{3A} - 1500 X_{2B}$ (Revenue - Cost)

Objective function, Maximize, $Z = 2000 X_{1A} + 1500 X_{1B} + 1000 X_{3A} - 1500 X_{2B}$

Subjected to the conditions , $(X_{1A} + X_{1B}) \leq 3500$,

$X_{2B} \leq 2000$,

$X_{3A} \leq 3000$,

$X_{1A} \leq 0.3 (X_{1A} + X_{3A})$,

$X_{3A} \leq 0.5 (X_{1A} + X_{3A})$

$X_{1B} \leq 0.5 (X_{1B} + X_{2B})$

$X_{2B} \leq 0.1 (X_{1B} + X_{2B})$

and $X_{1A}$, $X_{3A}$, $X_{1B}$ & $X_{2B} \geq 0$.

[19] A farmer has a 125 acre farm. He produces Radish, Lettuce and Potato. Whatever he raises is fully sold. He gets Rs. 5 per kg for radish, Rs. 4 per kg for Lettuce and Rs. 5 per kg for potato. The average yield per acre is 1500 kg for radish, 1800 kg for Lettuce and 1200 kg for potato. Cost of manure per acre is Rs. 187.50, Rs. 225 and Rs. 187.50 for radish, Lettuce and potato respectively. Labor required per acre is 6 man-days each for radish and potato and 5 man days for Lettuce. A total of 500 man-days of labor is available at the rate of Rs. 40 per man-day. Formulate this as an LPP model to maximize the profit.

Solution [19]:
Consider $X_R$, $X_L$ and $X_P$ denote quantity in ACRE land for Radish, Lettuce and Potato respectively.

Profit = Revenue - Total cost
Profit earned /acre = Profit/Kg * Yield /acre – manure cost – Labor
Profit from Radish = (5*1500 -187.5 – 6*40)
Profit from Lettuce = (4*1800 – 225 - 5*40)
Profit from Potato = (5*1200 – 187.5 – 6*40)

Total Profit = [(5*1500 -187.5 – 6*40) + (4*1800 – 225 - 5*40) + (5*1200 – 187.5 – 6*40) ]

Objective function:
Maximize $Z = 7072.5\, X_R + 6775\, X_L + 5572\, X_P$
Subjected to the constraints,

Land constraints - $(X_R + X_L + X_P) \leq 125$
Labor constraints - $(6X_R + 5X_L + 6\, X_P) \leq 500$
Also, $X_R, X_L \;\&\; X_P \geq 0$

[20] The vitamins V and W are found in two different foods, $F_1$ and $F_2$. The respective prices per unit of each food are Rs. 4 and Rs. 3. One unit of $F_1$ contains 2 units of vitamin V and 3 units of vitamin W. One unit of $F_2$ contains 4 units of vitamin V and 2 units of vitamin W. The daily requirements of V and W are at least 60 units and 75 units respectively. Formulate an LPP to meet the daily requirement of the vitamins at minimum cost.

Solution [20]:
Consider $X_1$ and $X_2$ denote quantity of Food F1 and F2 respectively.

Objective is to Minimize $Z = 4X_1 + 3X_2$

Subjected to the Constraints:
Requirement of V: $2X_1 + 4\, X_2 \geq 60$
Requirement of W: $3X_1 + 2\, X_2 \geq 75$
Non-negativity: $X_1$ and $X_2 \geq 0$

[21] A mutual fund has Rs. 2 million available for investment in Government bonds, blue chip stocks, speculative stocks and short-term bank deposits. The annual expected return and the risk factor are as shown.

| Investment | Return% | Risk factor (0- 100) |
|---|---|---|
| Bonds | 14 | 12 |
| Blue Chip | 19 | 24 |
| Speculative | 23 | 48 |
| Short-term | 12 | 6 |

The fund is required to keep at least Rs. 200,000 in short-term deposits and not to exceed an average risk factor of 42. Speculative stocks must not exceed 20% of the money invested. Formulate the LPP maximizing expected annual return.

Solution [21]:
Consider X1, X2, X3 and X4 denote the amounts invested in Govt. Bonds, Blue Chip, Speculative and Short-term respectively.

Maximize the Return on investments,
Maximize, $Z = 0.14 X_1 + 0.19X_2 + 0.23X_3 + 0.12X_4$

Subjected to the constraints,
Average risk factor = $[(12 x_1+24 x_2 + 48 x_3 + 6 x_4)]/[(x_1+ x_2+ x_3+ x_4)] \leq 42$.

⇨ This gives $30x_1+18x_2-6x_3+36x_4 \geq 0$.

Also $x_3 \leq 0.2 (x_1+ x_2+ x_3+ x_4)$ - Maximum limit on speculative stock not exceeding 20% of Money invested

⇨ This gives: $0.2 x_1 + 0.2 x_2 – 0.8 x_3 + 0.2 x_4 \geq 0$

LPP formulation is as follows:

Maximize, $Z = 0.14 X_1 + 0.19X_2 + 0.23X_3 + 0.12X_4$
Subjected to the constraints,

$30x_1+18x_2-6x_3+36x_4 \geq 0$ (Avg Risk factor)
$0.2 x_1 + 0.2 x_2 – 0.8 x_3 + 0.2 x_4 \geq 0$ (limit on speculative stock)
$x_1 + x_2+ x_3 + x_4 \leq 2,000,000$ (Available to Invest)
$x_4 \geq 200,000$ (Short Term Deposit)
also , $x_1, x_2, x_3, x_4 \geq 0$ (Non-negativity)

[22] Medical experts and dieticians opine that it is necessary for an adult to consume at least 75g proteins, 85g of Fats and 300g of Carbohydrates daily. The following table lists 6 types of food items and their respective nutritional values and the corresponding costs per Kg. Formulate the LP so that the total cost of food satisfying min. requirements of balanced diet is lowest

| Food Type | Food Type (Gms) per 100g | | | Cost/Kg(Rs) |
|---|---|---|---|---|
| | Proteins | Fats | Carbs | |
| 1 | 8 | 1.5 | 35 | **1** |
| 2 | 18 | 15 | | **3** |
| 3 | 16 | 4 | 7 | **4** |
| 4 | 4 | 20 | 2.5 | **2** |
| 5 | 5 | 8 | 40 | **1.5** |
| 6 | 2.5 | | 25 | **3** |
| **Min. daily requirements** | **75** | **85** | **300** | |

Solution [22]:
Consider $X_1$,$X_2$,$X_3$,$X_4$,$X_5$, and $X_6$ denote each FOOD types respectively to be used per day.
Objective Function is to minimize the cost of foods while meeting the minimum requirements of the nutrition.

**Minimize $Z = 1X_1 + 3X_2 + 4X_3 + 2X_4 + 1.5X_5 + 3{,}X_6$**
Subjected to the constraints,
Daily requirements of Proteins (75 gms)
→ $8X_1 + 18X_2 + 16X_3 + 4X_4 + 5X_5 + 2.5{,}X_6 \geq 75$
Daily requirements of Fats (85gms )
→ $1.5X_1 + 15X_2 + 4X_3 + 20X_4 + 8X_5 + 0{,}X_6 \geq 85$
Daily requirements of Carbohydrates (300gms )
→ $35X_1 + 0X_2 + 4X_3 + 2.5X_4 + 40X_5 + 25{,}X_6 \geq 300$
Also, $X_1$,$X_2$,$X_3$,$X_4$,$X_5$, and $X_6 \geq 0$



[23] A company manufactures two products X and Y, which require, the following resources. Which undergoes operation three machines M1, M2, and M3. The available capacities are 50, 25, and 15 hours respectively in the planning period. Product X requires 1 hour of machine M2 and 1 hour of machine M3. Product Y requires 2 hours of machine M1, 2 hours of machine M2 and 1 hour of machine M3. The profit contribution of products X and Y are Rs.50/- and Rs.40/-respectively

Solution [23]:
Consider $X_1$ and $X_2$ denote quantity of products X and Y respectively.
Objective is to Minimize $Z = 50X_1 + 40X_2$
Subjected to the Constraints:
Operation on M1 : $0X_1 + 2\,X_2 \leq 50$
Operation on M2 : $1X_1 + 2\,X_2 \leq 25$
Operation on M3 : $1X_1 + 1\,X_2 \leq 15$
Non-negativity: $X_1$ and $X_2 \geq 0$

[24] A retail store stocks two types of shirts A and B. These are packed in attractive cardboard boxes. During a week the store can sell a maximum of 400 shirts of type A and a maximum of 300 shirts of type B. The storage capacity, however, is limited to a maximum of 600 of both types combined. Type A shirt fetches a profit of Rs. 200/- per unit and type B a profit of Rs. 500/- per unit. How many of each type the store should stock per week to maximize the total profit? Formulate a mathematical model of the problem

Solution [24]:
Consider $X_1$ and $X_2$ denote quantity of Shirts A and B respectively.
Objective is to Maximize, $Z = 200X_1 + 500X_2$
Subjected to the Constraints:
Maximum sales of A, $X_1 \leq 400$
Maximum sales of B, $X_2 \leq 300$
Storage Capacity : $X_1 + X_2 \leq 600$
Non-negativity: $X_1$ and $X_2 \geq 0$

[25] A computer company manufactures laptops and desktops that fetch them a profit of Rs. 7000 and Rs. 5000 respectively. Each unit of Laptop takes 4 hrs to assemble and 2 hrs to test where as each unit of Desktop takes 3 hrs to assemble and 1 hr for testing. In a given month, the total assembly time available is 2100 hrs and total testing time available is 900 Hrs. Market can absorb only 2000 laptops and 3500 desktops. Formulate the LPP such that the profit is maximum

Solution [25]:
Consider $X_1$ and $X_2$ denote quantity of laptops and desktops respectively.
Objective is to Maximize, $Z = 7000X_1 + 5000X_2$

Subjected to the Constraints:
Maximum Assembly time, $4X_1 + 3X_2 \leq 2100$
Maximum Testing time, $2X_1 + 1X_2 \leq 900$
Market constraints : $X_1 \leq 2000$
Market constraints : $X2 \leq 3500$

Non-negativity: $X_1$ and $X_2 \geq 0$

[26] A garment manufacturer has a production line making two styles of shirts. Style I needs 200 g of cotton thread, 300 g of Dacron thread and 300 g of linen thread. Corresponding requirements of style II are 200g, 200g and 100g. The net contributions are Rs. 519.50 for style I and Rs. 515.90 for style II. The available inventory of cotton thread, Dacron thread and linen thread are, respectively, 240 kg, 260 kg and 220 kg. The manufacturer wants to determine the number of each style to be produced with the given inventory. Formulate the LPP model

Solution [26]:
Consider $X_1$ and $X_2$ denote quantity of Style I and Style II respectively.
Objective is to Maximize, $Z = 519.50X_1 + 515.90X_2$

Subjected to the Constraints:
Maximum Cotton, $0.2X_1 + 0.2X_2 \leq 240$
Maximum Dacron, $0.3X_1 + 0.2X_2 \leq 260$
Maximum Linen, $0.3X_1 + 0.1X_2 \leq 220$
Non-negativity: $X_1$ and $X_2 \geq 0$

Q1) Explain briefly explain the areas of management decision making, where OR techniques can be applied.
Q2) List the various phases of OR problems.
Q3) Define, i.) Feasible solution ii) Feasible region iii) optimal solution iv) Infeasible solution v) CPF solution vi) Degeneracy.
Q4) Define Operation Research.
Q5) Explain limitations of OR models.
Q6) Enumerate and briefly explain applications and limitations of OR to engineering problems.
Q7) Discuss the areas of management where operation research techniques are applied.
Q8) State the assumptions made in LPP and explain in brief any one of them.
Q9) Explain briefly the scope of Operation Research.
Q10) Describe the phases of OR.
Q11) Discuss the areas of managements where OR techniques are applied.
Q12) Give the classification of models used in OR. Explain the mathematical modeling process.
Q13) Explain the components involved in the formulation of LPP, with a simple example.
Q14) Explain with few points about Variables, Objective function, constraints and non-negativity



1. Solve by the following LPP by simplex method
   Maximize $z = 3x_1 + 2x_2$
   Subject to, $x_1 + x_2 \leq 8$
   $x_1 - x_2 \leq 2$
   $x_1 . x_2 \geq 0$
2. Solve the following LPP by simplex method:
   Maximize $z = 6x_1 + 8x_2$
   Subject to, $2x_1 + 8x_2 \leq 16$
   $2x_1 + 4x_2 \leq 8$
   $x_1 . x_2 \geq 0$
3. Use graphical method to solve LPP
   Minimize $z = 3x_1 + 5x_2$
   Subject to, $-3x_1 + 4x_2 \leq 12$----------- (1)
   $2x_1 + 3x_2 \geq 12$----------- (2)
   $2x_1 - x_2 \geq 0$----------- (3)
   $x_1 \geq 4$----------- (4)
   $x_2 \geq 2$----------- (5)
   $x_1 . x_2 \geq 0$
   Write the dual of the problem. 10 Marks

4. Solve the following LPP using simplex method
   Minimize $z = 2x_1 + 3x_2 + x_3$
   Subject to, $x_1 + 4x_2 + 2x_3 \leq 8$
   $3x_1 + 2x_2 \geq 6$
   $x_1 . x_2 \geq 0$

5. Solve the following LPP using dual simplex method
   Minimize $z = 2x_1 + x_2$
   Subject to, $4x_1 + 2x_2 \geq 6$

$x_1 + 2x_2 >\_3$
$x_1 . x_2 >\_0$

6. Solve the following LPP using Big M method
   Minimize $z = 2x_1 + 5x_2$
   Subject to, $x_1 + x_2 = 100$
   $x_1 <\_40$
   $x_2 >\_30$
   $x_1 . x_2 >\_0$
7. Using graphical method, Solve the LPP
   Maximize $z = 5x_1 + 4x_2$
   Subject to, $6x_1 + 4x_2 <\_24$
   $x_1 + 2x_2 <\_6$
   $-x_1 + x_2 <\_1$
   $x_1 . x_2 >\_0$
8. Using simplex method, Solve the following LPP
   Maximize $z = 4x_1 + 3x_2 + 6x_3$
   Subject to, $2x_1 + 3x_2 + 2x_3 <\_440$
   $4x_1 + 3x_2 <\_470$
   $2x_1 + 5x_2 <\_430$
   $x_1 . x_2 . x_3 >\_0$

9. Using graphical method, Solve the LPP
   Maximize $z = 3x_1 + 5x_2$
   Subject to, $x_1 <\_4$
   $2x_2 <\_12$
   $3x_1 + 2x_2 <\_18$
   $x_1 . >\_0$
   $x_2 >\_0$
10. Solve by simplex method
    Maximize $z = 3x_1 + 9x_2$
    Subject to, $x_1 + 4x_2 <\_8$
    $x_1 + 2x_2 <\_4$
    $x_1 . x_2 >\_0$
11. Solve the following LPP:
    Maximize $z = 3x_1 + 9x_2$
    Subject to, $x_1 + 4x_2 <\_8$
    $x_1 + 2x_2 <\_4$
    $x_1 . x_2 >\_0$